THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY IN NEW YORK.

THE one hundred and seventy-seventh regular meeting of the Society was held in New York City on Saturday, April 24, 1915, the following seventy-one members being in attendance at the two sessions:

Mr. J. W. Alexander, II, Professor M. J. Babb, Dr. Ida Barney, Dr. F. W. Beal, Dr. R. D. Beetle, Professor Susan R. Benedict, Mr. A. A. Bennett, Professor E. G. Bill, Professor G. D. Birkhoff, Professor Maxime Bôcher, Professor Joseph Bowden, Professor E. W. Brown, Dr. T. H. Brown, Professor A. B. Coble, Dr. Emily Coddington, Professor F. N. Cole, Dr. G. M. Conwell, Professor J. L. Coolidge, Professor Elizabeth B. Cowley, Dr. Louise D. Cummings, Dr. H. B. Curtis, Mrs. E. B. Davis, Professor L. P. Eisenhart, Professor H. B. Evans, Professor F. C. Ferry, Professor H. B. Fine, Dr. C. A. Fischer, Professor T. S. Fiske, Professor W. B. Fite, Professor A. S. Gale, Professor O. E. Glenn, Dr. G. M. Green, Professor C. C. Grove, Professor H. E. Hawkes, Professor E. V. Huntington, Dr. Dunham Jackson, Mr. S. A. Joffe, Professor Edward Kasner, Dr. L. M. Kells, Professor C. J. Keyser, Dr. E. A. T. Kircher, Professor W. R. Longley, Professor James Maclay, Dr. E. J. Miles, Professor H. H. Mitchell, Professor E. H. Moore, Dr. R. L. Moore, Mr. G. W. Mullins, Professor W. F. Osgood, Dr. Alexander Pell, Professor Anna J. Pell, Dr. G. A. Pfeiffer, Professor A. D. Pitcher, Dr. H. W. Reddick, Professor R. G. D. Richardson, Dr. R. B. Robbins, Professor W. H. Roever, Professor J. E. Rowe, Dr. Caroline E. Seely, Professor H. E. Slaught, Professor Clara E. Smith, Professor H. F. Stecker, Professor H. D. Thompson, Mr. H. S. Vandiver, Mr. C. E. Van Orstrand, Professor Oswald Veblen, Mr. R. A. Wetzel, Professor H. S. White, Miss E. C. Williams, Professor Ruth G. Wood, Professor J. W. Young.

Owing to the large amount of business to be transacted, the Council met on the evening of April 23, as well as on the following morning. President E. W. Brown occupied the chair at the Council meeting, being relieved at the sessions of

the Society by Ex-Presidents Osgood, Moore, and White, and Vice-President Veblen. The Council announced the election of the following persons to membership in the Society: Professor L. S. Hill, University of Montana; Miss G. I. McCain, Indiana University; Mr. J. F. Riley, Rice Institute. Eleven applications for membership in the Society were received. Professor P. F. Smith was reelected a member of the Editorial Committee of the *Transactions*, to serve for the three years beginning October 1, 1915.

The invitation of the Division of Mathematics of Harvard University to hold the summer meeting and colloquium of the Society at Harvard University in 1916 was accepted. A committee, consisting of Professors Osgood, Bôcher, Moore, White and the Secretary, was appointed to make the arrangements for this meeting. A committee was also appointed to prepare a list of nominations for officers and other members of the Council to be elected at the annual meeting. It was decided to hold the annual meeting this year in New York at such time as not to interfere with the meeting of the Chicago Section at Columbus in affiliation with the American association for the advancement of science.

Reports of committees were received concerning the proper attitude of the Society toward the movement against mathematics in the schools and the possible relations of the Society to the field now covered by the *American Mathematical Monthly*. In the former matter it was held to be inadvisable for the Society to take any action at the present time. Regarding the relations of the Society to the *Monthly*, the sense of the Council was embodied in the following resolution:

"It is deemed unwise for the American Mathematical Society to enter into the activities of the special field now covered by the *American Mathematical Monthly;* but the Council desires to express its realization of the importance of the work in this field and its value to mathematical science, and to say that should an organization be formed to deal specifically with this work, the Society would entertain toward such an organization only feelings of hearty good will and encouragement."

In the interval between the sessions the members took luncheon together. Thirty-seven members gathered in the evening for an enjoyable dinner at the Terrace Garden.

The following papers were read at this meeting:

(1) Professor C. J. DE LA VALLÉE POUSSIN: "Démonstration simplifiée d'un théorème de Vitali sur le passage à la limite sous le signe d'intégration."

(2) Dr. C. A. FISCHER: "Minima of double integrals with respect to one-sided variations."

(3) Dr. G. M. GREEN: "Nets of space curves."

(4) Dr. R. L. MOORE: "A set of axioms in terms of point, region, and motion."

(5) Dr. R. L. MOORE: "On the categoricity of a set of postulates."

(6) Mr. H. S. VANDIVER: "A property of cyclotomic integers and its relation to Fermat's last theorem. Second paper."

(7) Mr. J. F. RITT: "The reduction of invariant equations."

(8) Professor E. B. WILSON: "Linear momentum, kinetic energy, and angular momentum."

(9) Professor F. H. SAFFORD: "An irrational transformation of the Weierstrass $\wp$-function curves."

(10) Professor ARNOLD EMCH: "A certain class of functions connected with Fuchsian groups."

(11) Professor G. D. BIRKHOFF: "An elementary double inequality for the roots of an algebraic equation having greatest absolute value."

(12) Professor H. S. WHITE and Dr. LOUISE D. CUMMINGS: "Groupless triad systems on 15 elements" (preliminary report).

(13) Professor EDWARD KASNER: "Conformal geometry in the complex domain."

(14) Professor EDWARD KASNER: "Convergence proofs connected with equilong invariants."

(15) Professor E. V. HUNTINGTON: "Notes on the catenary, including the case of an extensible chain."

(16) Mr. R. C. STRACHAN: "Note on the catenary."

(17) Professor J. E. ROWE: "A property of the osculant conic of the rational cubic."

(18) Professor J. E. ROWE: "The node of the rational cubic as a rational curve in lines."

(19) Dr. F. W. BEAL: "A congruence of circles."

(20) Professor H. F. STECKER: "Linear integral equations whose solutions have only a finite number of terms."

(21) Mr. C. A. EPPERSON: "Note on Green's theorem."

(22) Mr. L. J. REED: "Some fundamental systems of formal modular invariants and covariants."

(23) Mr. J. R. KLINE: "Double elliptic geometry in terms of point and order."

(24) Dr. ALEXANDER PELL: "On the curves of constant torsion."

(25) Mr. J. H. WEAVER: "The Collection of Pappus" (preliminary report).

(26) Professor J. W. YOUNG and Dr. F. M. MORGAN: "The geometries associated with a certain system of Cremona groups."

(27) Professor T. H. GRONWALL: "A functional equation in the kinetic theory of gases."

(28) Mr. J. W. ALEXANDER, II: "A theorem on conformal representation."

Professor de la Vallée Poussin was introduced by Professor Osgood, Mr. Ritt by Professor Kasner, Mr. Strachan by Professor Huntington, Mr. Reed by Professor Glenn, Mr. Kline by Dr. R. L. Moore, and Mr. Weaver by Professor Babb. Mr. Epperson's paper was communicated by Professor G. C. Evans. In the absence of the authors the papers of Professor Wilson, Professor Safford, Professor Emch, Mr. Epperson, Professor Young and Dr. Morgan, Professor Gronwall, and Mr. Alexander were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Professor de la Vallée Poussin presented a new proof of a theorem due to Vitali. This proof forms part of a paper entitled "Les intégrales de Lebesgue" which will appear in the *Transactions* of the Society at a later date.

2. In some problems in minima of double integrals, the surface over which the integral is taken is restricted to lie in a given closed region R. Then it may happen that there is no extremal surface bounded by a previously given space curve which lies entirely within R, but that there is a surface consisting of a part of an extremal surface and a part of the boundary of R which minimizes the given integral. In this paper Dr. Fischer derives some necessary conditions for such a minimum, and also a set of sufficient conditions.

3. In the metric differential geometry of a surface, use is frequently made of the Gaussian parameter form of representa-

tion, the surface being thereby referred to an arbitrary net of parameter curves. But only the organic properties of the surface, and not those of the net, are the objects of study. In this paper Dr. Green investigates an arbitrary net of curves for its own sake. A canonical development for the configuration in the neighborhood of a point is found, and leads to a consideration of certain osculating quadric and cubic surfaces. Of especial interest and importance is a certain congruence, formed by the lines of intersection of the osculating planes to the curves of the two families of the net at the points of the surface. The same congruence was used in obtaining the canonical development of a surface referred to a conjugate net, given in the report of the February, 1914, meeting of the Society. The study of this congruence leads to certain theorems concerning the general theory of congruences, in which the focal points on a line of the congruence appear as the double points of an involution. New theorems of a metric nature, concerning geodesics and normal congruences, are corollaries of the general theory. The entire discussion is founded on the consideration of a completely integrable system of partial differential equations, following out the general ideas of Wilczynski.

4. Dr. Moore proposes the following set of axioms for plane analysis situs in terms of point and region.

1. A region is a set of points.

2. Every region contains at least two points.

Definitions. P is a limit point of M if every region which contains P contains at least one point of M different from P. A set of points is connected if, however it be divided into two subsets, one of them contains a limit point of the other one. The boundary of a region R is the set of all those limit points of R that do not belong to R. If R is a region, R' denotes R plus its boundary.

3. There exists a countably infinite set of regions R_1, R_2, $R_3 \cdots$ such that (1) if P and X are distinct points within a region R, then there exists n_{PR} such that if $n > n_{PR}$ and R_n contains P then R_n' is a proper subset of $R - X$; (2) if $\delta > 0$ and P is a point, then there exists n greater than δ such that R_n contains P.

4. Each region is connected and so is its exterior.

5. If two regions contain a point in common, then they

also contain in common a region which contains that point.

6. If the point P does not belong to the region R, then every region containing P contains a region which has no point in common with R.

7. Not every infinite set of points has a limit point.

8. If A and B are distinct points on the boundary of a region R and $A \times B$ is a Jordan arc whose interior t lies entirely within T (where T is either R or its exterior) and $T - t = S_1 + S_2 + S_3$, where S_1, S_2, S_3 are mutually exclusive point sets, then one of the sets S_1, S_2, S_3 contains a limit point of one of the other two.

9. The Heine-Borel proposition as applied to a set of regions covering a given region and its boundary.

10. If R is a region, T either its interior or its exterior, B a point in T or on the boundary of R, A a point in T, and $A \times B$ a Jordan arc whose interior t lies in T, then if T is connected so is $T - t$.

11. Every Jordan arc is the boundary of only one region.

12. If one point of a region R is interior to a Jordan curve lying in R, then every point of R is interior to such a curve.

From axioms 1–12 follows a considerable body of propositions relating to non-metrical properties of plane curves and point sets. It follows from axioms 1–6 and 9 that every two points of a region can be joined by a Jordan arc lying in that region. If axiom 12 is omitted and in place of axiom 3 are substituted certain axioms involving the third undefined term motion, there results a system of axioms categorical with respect to point, motion, and limit point of a point set as defined above.

5. In a recent paper* Dr. Moore has given a system of axioms for the linear continuum in terms of point and limit. In his second paper Dr. Moore proposes to show that, while this set of axioms is categorical with respect to point and order as there defined, nevertheless it is not categorical with respect to point and limit but will become so if for axioms 5 and 8 are substituted the following axioms 5′ and 8′:

5′. If r_1 and r_2 are two mutually exclusive rays and if the set S of all points contains points not belonging to $r_1 + r_2$, then

* "The linear continuum in terms of point and limit," *Annals of Mathematics*, vol. 16 (March, 1915), pp. 123–133.

every infinite set of points lying in $S - (r_1 + r_2)$ has at least one limit point.

8′. If P is a limit point of M, then P is a limit point of every infinite subset of M.

6. In a paper presented at the January meeting of the Society, Mr. Vandiver proved a relation in connection with Fermat's last theorem from which the criteria of Kummer and Furtwängler were deduced. The present paper contains developments along the same line.

7. In computing invariants under a group by executing a transformation of the group and then eliminating the parameters of the transformation, one obtains a system of equations which, when properly separated, yield the invariants. It seems that the only proof heretofore known of the separability of the invariant equations consists in an appeal to the results of the Lie theory. Mr. Ritt gives a direct method for replacing the invariant equations by a system of invariants.

8. Professor Wilson points out that some persons, who should know better, seem to think that the *kinematic* resolution of motion of laminas and rigid bodies into a velocity of translation of any arbitrary point and an angular velocity about an axis through that point, is *dynamically* valid. As a matter of fact the center of mass is introduced in kinetics precisely so that we may have a point for which the kinematic resolution holds dynamically. A number of simple geometric theorems develop from the search after points other than the center of mass for which dynamic results have the same simple resolution as the kinematic. The paper will appear in the *American Mathematical Monthly*.

9. On September 10, 1912, Professor Safford read before the Society a preliminary communication entitled "An irrational transformation of the Weierstrass $\wp$-function curves." The present paper gives the final results and will be sent to the *Archiv der Mathematik und Physik*, in which the first paper appeared on June 12, 1914. The original curves are obtainable from $x + iy = \wp(t + iu)$, and are of the fourth degree. After transformation and rationalization the resulting equation is of the sixteenth degree, but is resolvable into three factors,

corresponding to one irresolvable curve of the eighth degree and one of the fourth degree taken twice.

10. In this paper Professor Emch shows that the series

$$\sum_{\lambda=0}^{\infty}[R\{(z_\lambda - a_\lambda)(z_\lambda - b_\lambda)\} - R\{(z_\lambda' - a_\lambda')(z_\lambda' - b_\lambda')\}],$$

extended over a Fuchsian group with the unit circle as a natural boundary, where z, z', a, b, a', b' are within and not on the unit circle, z', a', b' are fixed, and $R(z)$ is a rational function, under certain restrictions represents an analytic function of z with the unit circle as a natural boundary. It appears that, in general, these functions are not automorphic in the ordinary sense of the word.

11. The note by Professor Birkhoff appears in the present number of the BULLETIN.

12. From previous publications and a paper presented to this Society in October, 1914, 44 different triad systems on 15 elements are known. These 44 belong to groups of order from 8!/2 down to 2. Systems whose group is identity, or groupless systems, on 15 elements have not been known hitherto. This paper of Professor White and Dr. Cummings exhibits a mode of deriving new systems from old, and applies it to the derivation of groupless systems. Over 20 such are announced, and the extent of the research now under way is indicated.

13. Real conformal geometry deals with the ∞^2 real points of the usual Gaussian plane under real conformal transformation. Complex conformal geometry, as discussed by Professor Kasner, deals with the totality of ∞^4 complex points of the plane under the larger group of complex conformal transformations. In earlier papers (see Proceedings of the International Congress of Mathematicians, Cambridge, volume 2, pages 81–87) it was shown that in the real plane a regular analytic curvilinear angle has an invariant of higher order only when its magnitude is a rational part of 360° (including the horn angles of magnitude 0). The present paper includes a complete classification of analytic angles in the complex plane. In contrast with the real discussion, it is shown that certain

types of angle have an infinite number of higher invariants, and other types are capable of uniquely determining a conformal transformation. The bisection problem (defined in the paper cited) is also generalized and it is shown that the number of solutions may be not only 0, 1, 2, ∞, as in the real case, but also 3, 4, or any finite number.

14. In the paper cited above, Professor Kasner discussed also the invariants of a pair of analytic curves having a common tangent under the group of equilong transformations of the plane. It was shown that each type of configuration (classified according to the order of contact of the two curves) has a unique invariant J_{2k+1}. Hence if two configurations of the same type have the same J it follows that a transformation converting the one into the other can be formally calculated as a power series. The author now completes the theory by showing that these series are always convergent. This is done by reducing the problem to a differential equation of the form $dy/dx = P/Q$, where P and Q are series in x and y without absolute terms. The regular Cauchy theory is thus not applicable, but it is shown that after certain substitutions the Briot-Bouquet criterion may be used. The equilong theory thus turns out to be essentially simpler than the (roughly) dual conformal theory, since in the latter case the convergence question cannot be reduced to differential equations.

15. Professor Huntington's paper shows how to compute the length of a chord of a generalized catenary from the following data: the inclination of the chord to the horizontal; the weight per unit length, area of cross-section, and modulus of elasticity of the tape; the nominal length of the arc; and either the tension at the upper end or the tension at the lower end; and tables are provided for facilitating the computation. The paper also contains tables giving an auxiliary variable z in terms of the arguments $(\cosh z - 1)/z$ or $z/\cosh z$ or $\sinh z/z$, these tables being useful in the ordinary computations connected with the common catenary.

16. Mr. Strachan points out the desirability of having tables for the quantities $l = u/\sinh u$ and $k = 2u/(\cosh u - 1)$ for use in certain well-known computations connected with the catenary, and suggests the following form for such a table:

$\theta = gd\, u$	u	$\sinh u$	$\cosh u$	$\tanh u$	l	k

where the argument θ runs from 0° to 90° at intervals of 20″, and the functions are tabulated to six figures.

As an alternative, he suggests the following form, to be used in connection with existing trigonometrical tables:

$\theta = gd\, u$	$\log u$	$\log l$	$\log k$

17. In Professor Rowe's first paper the osculant conic of the rational cubic is defined algebraically, and it is proved not only that this osculant conic at a particular point whose parameter is t_1 touches the R^3 at that point (which is a well-known theorem), but that the parameters of the remaining intersections of the R^3 and the osculant conic constitute a self-apolar set.

18. In this paper Professor Rowe shows how to derive from the parametric equations of the rational cubic the equation of the node in line coordinates. The parametric equations of the node in lines may be found by taking the polar of the flex cubic as to the line equation of the curve.

19. Dr. Beal considers a congruence of circles which he calls a congruence C and which may be defined in the following manner: Any point P on a circle of the two-parameter system is to be a point P of a surface Σ whose tangent plane at P passes through the center of the circle and cuts the plane of the circle under an angle which is a function of the circle alone. Of particular interest is a congruence C such that the center of the circle is the point at which the plane of the circle touches its envelope M. In this case the surfaces Σ have the same total curvature as surface M. The surfaces Σ are not such that they admit a congruence C_1 such that M is one of an infinity of surfaces related to C_1 in the same manner as Σ is to C, unless M is pseudospherical. Then the surfaces Σ are of course Bäcklund transforms of M.

20. In solving actual problems of linear integral equations the Fredholm series $D(s, t, \lambda)$ and $D(\lambda)$ often have only a finite number of terms. It is the purpose of Professor Stecker's paper to determine the conditions under which this takes place. It is shown that it is necessary and sufficient

that the kernel be of the form $\sum_{i=1}^{i=n} \varphi_i(s)\psi_i(t)$, except, possibly, at the points of a set of content zero; or that the known term in the integral equation itself satisfy a certain integral equation. The corresponding forms of solution of this linear integral equation are also obtained.

21. The aim of Mr. Epperson's paper is to extend Green's theorem to apply to integro-differential relations equivalent to the general linear partial differential equation of the second order in two variables. These relations are written in such a way as not to involve derivatives of the second order, and the theorem is proved without assuming their existence. This point of view is desirable for the possibility of its application to physics. The proof is deduced without making use of Green's theorem in its usual form.

22. The method of constructing formal modular concomitants of a form that Mr. Reed uses in this paper is the one outlined by Professor O. E. Glenn in an article published in the BULLETIN for January, 1915. The method consists in finding transvectants of the form with the absolute covariants of the modular group and in the use of certain invariant operators. By this method are constructed fundamental systems for the linear form modulo p, for the quadratic form modulo 2, and a simultaneous system for the linear and the quadratic form. These three systems are shown to be complete. The same method applied to the quadratic form modulo p gives a series of irreducible concomitants, $\frac{1}{2}(p+7)$ in number.

23. In his Rational Geometry, Halsted gives a set of axioms for double elliptic plane geometry in terms of point, order, association, and congruence. Mr. Kline proposes the following categorical set of eleven axioms for three dimensional double elliptic geometry in terms of point and order alone:

1. If A is a point, there exists at least one point A', different from A, such that the order $AA'C$ does not hold for any point C. A' is called an opposite of A.

2. There exist at least two distinct points A and B such that $B \neq A'$.

3. If ABC, CBA.

4. If ABC and A' is any opposite of A, $A'CB$.

5. If ABC and ACD, then not ADB.

6. If ABC and BCD, then either ABD or $AD'B$.

7. If there exist an infinite number of points, there exist three points A, B, D such that D is not on the line AB.

8. If AED and ABF and D is not on AB, then there exists an H such that BHD and EHF.

9 and 10 are the same as Axioms IX and X of Veblen's euclidean system.

11. The Dedekind cut postulate for every segment. The straight line AB is defined as the set of all points X satisfying one of the eight conditions $X = A$, $X = B$, $X = A'$, $X = B'$, XAB, AXB, ABX, $AX'B$.

It follows from Axioms 1–6 that a point has only one opposite; from Axioms 1 to 6 and Axiom 8 follows the existence of an infinite number of points and the fact that any two distinct points of a line, not opposites, determine this line. That the points of a line are in cyclical order follows from Axioms 1 to 6 and Axiom 8, Axiom 8 being merely used to show that, assuming the hypothesis of Axiom 5 satisfied, D is on the line AB. That every two lines in a plane intersect in two points, which are opposites, is a consequence of Axioms 1–8. It follows from Axioms 1 to 10 that every two planes intersect in a straight line.

24. Dr. Pell shows that the coordinates of a curve of constant torsion can be expressed as the sum of the coordinates of a minimal curve and of a spherical curve. The curve of constant torsion lies on a developable surface whose edge of regression is the corresponding minimal curve, the distance between corresponding points being constant. Some properties of a circular helix are generalized for curves of constant torsion.

25. Mr. Weaver has made a translation of the Collection of Pappus, to which he is adding a commentary, explaining in modern form the Greek geometric methods. In this preliminary paper he discusses the various criticisms of Pappus in the light of this extended study. He gives references from Pappus's own works, which have doubtless been missed by many historians who have blamed Pappus for literary theft or on the other hand have given credit which Pappus definitely disclaims. These ancient impressions being corrected, Mr.

Weaver wishes to discuss in succeeding papers some notions developed by Pappus which he believes are not generally known.

26. The paper by Professor Young and Dr. Morgan is a revision from a much more general point of view of a paper presented by them to the Society two years ago (April 26, 1913) under the title "The geometry associated with a certain group of Cremona transformations in space." This earlier paper called attention to a system $G_1, G_2, G_3, \cdots, G_n, \cdots$ of groups of Cremona transformations (of order n) in spaces of $1, 2, 3, \cdots, n$ dimensions respectively, and developed elementary properties of the geometry associated with one of the particular forms assumed by the group G_3 in space of three dimensions. In the present paper the symbol G (without subscripts) is used to denote any one of the groups G_i ($i = 1, 2, \cdots, n$); the associated point space, on which the transformations of G operate in a one-to-one reciprocal way, is denoted by S. The geometry of the system (S, G) is then developed. The theorems are for the most part the same as in the previous paper; the novelty in the present revision lies merely in the recognition (explicitly) of the fact that the theorems are of "general reference" in the infinite system of geometries defined by the groups G_i.

27. Let ξ, η, ζ, and ξ_1, η_1, ζ_1 be the velocity components of two spherical molecules at the instant of their collision, and $\varphi(\xi, \eta, \zeta)$ the logarithm of the function defining the distribution of velocities; then the Maxwell-Boltzmann fundamental theorem leads to the functional equation

$$\varphi(\xi, \eta, \zeta) + \varphi(\xi_1, \eta_1, \zeta_1) = f(\xi + \xi_1, \eta + \eta_1, \zeta + \zeta_1, \xi^2 + \eta^2 + \zeta^2 + \xi_1^2 + \eta_1^2 + \zeta_1^2).$$

Assuming the existence of all six partial derivatives of the second order of φ, it is readily shown (Boltzmann, Vorlesungen über Gastheorie, volume 1, pages 128–131) that the general solution of the functional equation is

$$\varphi(\xi, \eta, \zeta) = a + b_1\xi + b_2\eta + b_3\zeta + c(\xi^2 + \eta^2 + \zeta^2)$$

with constant coefficients. In the present paper, Professor Gronwall establishes the same result under the sole assumption of the continuity of φ for all real values of the variables.

F. N. Cole,
Secretary.

AN ELEMENTARY DOUBLE INEQUALITY FOR THE ROOTS OF AN ALGEBRAIC EQUATION HAVING GREATEST ABSOLUTE VALUE.

BY PROFESSOR GEORGE D. BIRKHOFF.

(Read before the American Mathematical Society, April 24, 1915.)

LET there be given an algebraic equation of the nth degree written

$$x^n + C_{n,1}a_1x^{n-1} + C_{n,2}a_2x^{n-2} + \cdots + a_n = 0,$$

where $C_{n,1}$, $C_{n,2}$, $\cdots$ denote binomial coefficients. Let x_1, x_2, $\cdots$, x_n denote its roots, and X the greatest absolute value of any of these roots.

From the equations which express the coefficients in terms of the roots

$$C_{n,k}a_k = (-1)^k \Sigma x_1x_2\cdots x_k \quad (k = 1, 2, \cdots n)$$

we infer at once that

$$|a_k| \leqq X^k.$$

In fact there are precisely $C_{n,k}$ terms on the right-hand side of the kth equation, each of which is not greater than X^k.

Hence, if α stands for the greatest of the quantities

$$|a_1|, |a_2|^{1/2}, \cdots, |a_n|^{1/n},$$

it is clear that we have $X \geqq \alpha$.

Also from the given algebraic equation we obtain directly

$$|x|^n \leqq C_{n,1}|a_1||x|^{n-1} + \cdots + |a_n|,$$

where x represents any of the quantities x_1, x_2, $\cdots$, x_n. Replacing $|x|$ by one of its possible values X, and each quantity $|a_k|$ by the quantity α^k, at least as great, we obtain

$$|X|^n \leqq C_{n,1}\alpha|X|^{n-1} + \cdots + \alpha^n,$$

which may be written

$$X^n \leqq (X + \alpha)^n - X^n.$$

Transposing the term X^n to the left-hand side and extracting the nth root of the two members of the resulting inequality, we find

$$X \leqq \frac{\alpha}{2^{1/n} - 1}.$$

The roots of largest absolute value X are restricted by the double inequality

$$\alpha \leqq X \leqq \frac{\alpha}{2^{1/n} - 1},$$

where α denotes the largest of the quantities

$$| a_1 |, | a_2 |^{1/2}, \cdots, | a_n |^{1/n}.$$

The inequality $X \geqq \alpha$ was noted by R. D. Carmichael and T. E. Mason,* who observed also that the lower limit is reached if the equation is

$$(x + \alpha)^n = 0.$$

It is also evident that the upper limit found above is reached if the equation is

$$2x^n - (x + \alpha)^n = 0.$$

HARVARD UNIVERSITY,
April 23, 1915.

CERTAIN NON-ENUMERABLE SETS OF INFINITE PERMUTATIONS.

BY PROFESSOR A. B. FRIZELL.

(Read before the American Mathematical Society April 10 and December 28, 1914.)

1. THE simplest element of a permutation is the pairing of one of the objects permuted with a number indicating its place in the permutation. Such a pairing may be called a *primitive element* and denoted by (i, n), where n is the object and i the number of the place assigned to it. In this paper the objects will all be numbers, finite or transfinite.

2. Permutations of finite sets are simply collocations of primitive elements. They are conveniently denoted by ex-

* BULLETIN, vol. 21 (1914), pp. 14–22; in particular p. 20.

pressions of the form $(i_1, n_1)O(i_2, n_2)O \cdots O(i_k, n_k)$ $(i_1 < i_2 < \cdots < i_k)$ and may be said to form *polynomial* elements. The number of the place will be called the *index* of a term (or monomial element) and the number placed there its *digit*. Index and digit may be transfinite and the terms may form an infinite series instead of a finite polynomial.

3. If in an infinite well ordered set only a finite number of objects are permuted, the pairing of these objects with their new places may be called a *finite permutation*. It is easy to see that any set of finite permutations can be well ordered. If, e.g., in the series 1, 2, $\cdots$ we bring the numbers 2, 3, $\cdots$ successively to the first place, leaving the rest unchanged, and in each resulting series do likewise for the second place, then similarly for the third, and so on, the whole set is well ordered by the process of formation.

4. By an *infinite permutation* is to be understood here a pairing whereby infinitely many objects change their places. Thus, for example

$$(1, 2)O(2, 1)O(3, 4)O(4, 3)O \cdots$$

is an infinite permutation of the series 1, 2, $\cdots$ yielding the new series 2, 1, 4, 3, 6, 5, $\cdots$. Another example is obtained by interchanging the powers of 2 with the corresponding powers of 3, giving the series 1, 3, 2, 9, 5, 6, 7, 27, 4, 10, 11, $\cdots$.

5. The polynomial elements that can be made with an ω-series of digits and an ω-series of indices form an enumerable set, since they are in one-to-one correspondence with the set of algebraic polynomials in which all coefficients are natural numbers. Let us order these polynomials according to the rules

(1) $(i, m) < (i, n)$ if $m < n$;

(2) $(i_1, m) < (i_2, n)$ if $i_1 < i_2$;

(3) $p < q$ if p has fewer terms than q;

(4) $p < q$ if they have the same number of terms but the first term of p that is not in q is lower than the first term of q which is not contained in p.

It is not difficult to show inductively that in this way the set is well ordered. Its type of order is the same as that which results from arranging the natural numbers according

to their prime factors—first simple primes, then products of two primes, of three primes, and so on. This observation also proves at once that the set is countable.

6. The polynomial elements $\Sigma(i, n)$ are now to be used in forming infinite permutations. In the series 1, 2, $\cdots$ let the monomial (i, n) interchange an ω-series of products of n primes with a similar series of products of i primes, and let the polynomial $\Sigma(i, n)$ effect simultaneously the exchanges indicated by its several terms. Thus the first primitive (1, 1) is an identical transformation, leaving the series unchanged; the second monomial (1, 2) is to interchange the simple primes with some ω-series of products of two primes; let it be the first one, $2p_i$, and we get the permutation 1, 4, 6, 2, 10, 3, 14, 8, 9, 5, 22, 12, 26, 7, 15, 16, 34, 18, 38, 20, $\cdots$. The transformation numbered ω is (2, 1); let it exchange the simple primes and the series $3p_i$. The next transformation is (2, 2); let it interchange the series $2p_i$ and $3p_i$, yielding the series 1, 2, 3, 9, 5, 15, 7, 8, 4, 21, 11, 12, 13, 33, 6, 16, 17, 18, 19, 20, 10, $\cdots$. We have, then, an enumerably infinite set of infinite permutations, since to every polynomial has been assigned a finite set of transpositions as just described. Whether these transpositions are all used does not interest us.

7. It is now easy to obtain non-denumerable sets by repeating the above process. To illustrate, let (1, 2) operate on the series of primes in the result of the transformation (2, 2); it produces the series 1, 2, 7, 9, 13, 15, 3, 8, 4, 21, 29, 12, 5, 33, 6, 16, 43, 18, 19, 20, 10, 39, $\cdots$. Let $p_i^{(0)}$ denote a polynomial and $P_i^{(0)}$ the result of applying it as in § 6, where i is the ordinal symbol assigned to $p_i^{(0)}$ by the rules of § 5, and let the ω-series used in § 6 be numbered likewise. Then $P^{(1)} \sim (i, n)$ shall be the permutation obtained by applying $p_i^{(0)}$ as a transformation to the series whose ordinal symbol is n. The new symbols (i, n) are to be combined into new polynomials $p^{(1)}$ as in § 2 and ordered by the rules of § 5. The new set of permutations $P^{(1)}$ cannot be put into one-to-one correspondence with set $[P^{(0)}]$. For on this assumption there still exists a $P^{(1)}$ which differs from $P_1^{(1)}$ in the transformation $p^{(0)}$ performed on its first ω-series, differs at the same time from $P_2^{(1)}$ in the transformation performed on its second ω-series, and from $P_n^{(1)}$ in that performed on its nth ω-series (where n runs through the values 1, 2, $\cdots \omega$, $\cdots, \omega^2, \cdots$), and is therefore not contained in the assumed list, which is a contradiction. Hence $[P^{(1)}]$ is not enumerable.

8. It is worth while to study the preceding types of order in a different way, following Cantor; they are all comprehended under the concept of a "Belegung." In forming the primitive elements of the $p^{(0)}$, two places are covered with an ω-series of numbers; this covering is denoted by ω^2. For the binomials, four places i_1, n_1, i_2, n_2 are covered with an ω-series, indicated by ω^4. The polynomials of N terms have $2N$ places to cover, giving the ordinal symbol ω^{2N}, whence the whole set $[p^{(0)}]$ is of ordinal type ω^ω. And this is the same as the type of the set of ω-series of prime factors in the system of natural numbers, for the products of two primes exhibit a single ω-series of ω-series, $2p_i, 3p_i, 5p_i, \cdots$; in the products of three primes $p_{i_1} \cdot p_{i_2} \cdot p_i$ the successive ω-series are obtained by covering two places i_1 and i_2 with an ω-series, with four factors three places are covered, and so on. The same type again is exhibited in each $P^{(0)}$ (cf. § 5) and therefore in forming the $P^{(1)}$ we have likewise a covering. In the $P^{(1)}$ each ω-series may be re-arranged as an ω^ω-series and this process admits an ω-series of repetitions, but no more; beyond this point it yields nothing new.

There is a bit of formalism in the preceding which perhaps calls for closer scrutiny. The set of products of N primes is in the strict sense a covering of N factors with an ω-series of values. Think of a row of N boxes or spaces into each of which a compositor throws a prime. The totality of different ways in which he might set up this line is precisely what is meant by the symbol ω^N. The symbol ω^ω, then, would mean, by analogy, that to every one of an ω-series of spaces is assigned arbitrarily a prime number. But if an ω-series of places is covered in this way with the series of primes we certain y do not get the set of natural numbers. And the covering of an ω-series with an ω-series is not a "limit" of the coverings ω^N, because the set $[p^{(0)}]$ lies between them. The use of ω^ω for this set is merely a convenient symbolism not capable of supporting any logical deductions.

9. The process by which the permutations $P^{(1)}$ were obtained from the $P^{(0)}$, however, being carried out by finite arithmetical steps, is not liable to inconsistency and, moreover, can be repeated. Thus we get a new set $[P^{(2)}]$, the digit n running through the series of all ordinal values lower than the ordinal type of the set of series in $P^{(1)}$. Suppose that to every $P^{(2)}$ has been assigned a $P^{(1)}$. There exists a $P^{(2)}$ from which

every $P_\nu^{(2)}$ differs in its νth transformation (i_ν, n_ν) for values of ν forming a series of type $> \omega$. Hence the terms in the corresponding polynomial $p^{(2)}$ are a set whose type is $> \omega$. But there are no such polynomials. Therefore the $P^{(2)}$ cannot be put into one-to-one correspondence with the $P^{(1)}$. The same reasoning holds for the $P^{(1)}$, so we have two proofs that this set is non-enumerable.

10. The process of § 7 applied to the permutations $P^{(2)}$ yields a new set $[P^{(3)}]$ and the reasoning of § 9 shows that this set is not equivalent to $[P^{(2)}]$. Therefore by strict induction from N to $N+1$ we infer the existence of an ω-series of sets of infinite permutations no one of which can be put into one-to-one correspondence with its predecessor. Ordinally $[P^{(N+1)}] > [P^{(N)}]$ for $N = 0, 1, 2, \cdots$. In Cantor's terminology, the set of infinite permutations of a simple infinity of objects presents an ordinal type higher than any finite aleph.

McPherson, Kansas.

GEORGE WILLIAM HILL, 1838–1914.*

George William Hill was the son of John William Hill and Catherine Smith, and was born in New York City on March 3, 1838. Both his father and grandfather were artists and he himself was of English and Huguenot descent. His early education like that of most of the men of his time in America gave him few advantages. In 1846, when his father moved from New York to the farm at West Nyack, the country was too busy with material development to produce many teachers who could give any but the most elementary instruction, and the country school which he attended must have been inferior in this respect to those of the larger cities. Even at Rutgers College in New Jersey, to which Hill was sent owing to the exhibition of unusual capacity and from which he took his degree in 1859, the course probably went but little beyond that now found in secondary schools. There, however, he came under the influence of a man whose ideas on education were unusual. Dr. Strong, according to Hill's evidence, believed only in the classic treatises; but little published after

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1840 was admitted to his library. Hill's sound knowledge of the fundamentals of his subject is doubtless due to this course of reading.

Hill's first paper, published in 1859 when he was but twenty-one years of age and before he had taken his degree at college, is a half-page note on the curve of a drawbridge. Two years later he showed his capacity in the essay which gained a prize offered by *Runkle's Mathematical Monthly* for the best solution of a problem connected with the constitution of the Earth. President R. S. Woodward, who has himself worked much at this subject, says that the memoir is still worthy of careful reading.

In the year 1861 he joined the staff of the Nautical Almanac office which then had its headquarters in Cambridge, Mass., and for a year or two he worked there and thus had an opportunity for association with some of the ablest men of the time in astronomical science. But he soon obtained permission to do his work at the home in West Nyack which he never seemed to leave willingly during the rest of his life. It was there that nearly all his best work was done. In fact he was only away from it for one considerable period and this is covered by his residence in Washington from about the year 1882 until 1892; even during that time the summers were generally spent in West Nyack.

In the first ten years after leaving college, Hill only published eight papers and none of them deal with celestial mechanics in the modern sense of the term. But from his output after that time it is evident that he had been reading and digesting the newer treatises and memoirs as they appeared. Delaunay's two magnificent volumes on the lunar theory were published in 1860 and 1869, respectively, and the methods of that investigator exercised a fascination over Hill for the rest of his life. The other great lunar theorist of the period, P. A. Hansen, had been explaining his methods for many years before this time and Hill was probably one of the few men of his time who understood them thoroughly. He does not seem to have been particularly drawn to them, although they are used in his theories of Jupiter and Saturn with but little alteration. It is difficult to find many traces of other influences in his work. His most celebrated memoir, it is true, is based on one of Euler's numerous methods, as he himself tells, but after the start he proceeds entirely on lines of his own devising.

The publications which follow his first attempts during this early period exhibit knowledge of theoretical astronomy and the power to handle large masses of numbers rather than any unusual mathematical ability. In his discussion of the observations of the great comet of 1858 which was undertaken to obtain a satisfactory orbit (1867), he has to deal with 363 places gathered from many sources. As usual with Hill, he does not confine himself to the main point but discusses systematic errors between different observatories and those due to the size of telescope used. His final conclusion is that there is no evidence of any force other than gravitation influencing the motion of the comet.

It is probable that his work on this body was responsible for the next three papers: on the reduction of star places, the determination of the elements of a circular orbit and the conversion of latitudes and longitudes into right ascensions and declinations, or at any rate that it drew his attention to these fundamental problems. But he was soon to lay them in the background for more original investigations in celestial mechanics proper. One can see in his published work the gradual approach to this subject. His tenth memoir is a correction to the elements of the orbit of Venus from observations extending over 33 years. It is followed by a derivation of the mass of Jupiter from the perturbations of certain asteroids, and the calculation of an inequality of very long period in the motion of Saturn. Shortly before, however, he had been assisting in the campaign which had started some years earlier to get the utmost out of the transits of Venus in 1874 and 1882. Part II of the Papers of the U. S. Commission relating to the transits is by his hand; it consists of charts and tables for facilitating predictions of the several phases at any place on the globe.

The active period of Hill's work in celestial mechanics began in 1872. Between that year and 1877, when his two chief memoirs appeared, he published eleven papers on various phases of the subject besides seven others in pure and applied mathematics and the long transit of Venus calculations already mentioned. Most of them are quite brief and call for no special mention.

In order that the value of Hill's contributions to celestial mechanics and more particularly to the lunar theory may be made clear, it is necessary to say a few words as to the con-

dition of the subject at the time they were published. For two hundred years mathematical astronomers, many of them of the first rank, had been devoting their energies to furnishing a complete demonstration of the power of the law of gravitation to account for the motions of all the bodies in the solar system within the degree of accuracy of the observations. In the third quarter of the nineteenth century it was evident that this demonstration would soon be made. Leverrier was publishing his tables for the positions of the great planets, while Hansen and Delaunay had completed their work on the moon. For the purposes of navigation all needed accuracy had been obtained, and from the scientific side there seemed to be but few matters which needed explanation: the final polish which a few industrious workers might give was the last step. There was thus danger that the subject of celestial mechanics might encounter a blank prospect. The number of investigators began to dwindle. At the same time, pure mathematics and physics were showing vast territories to be explored, while the discovery of spectrum analysis and the use of the photographic plate attracted many astronomers who earlier would have devoted themselves to the mathematical side of the subject. From the old point of view this attitude on the part of astronomers was justifiable. But Hill saw that there were problems other than the mere verification of the law of gravitation by comparisons of theory and observation of the chief bodies in the solar system which would demand solution. He also saw, partly from the industrious work of Newcomb on the old and modern observations of the moon, that even the enormous labors of Hansen and Delaunay on the theory of its motion would demand extension and verification if a test of the Newtonian law to the degree of accuracy of the observations were required. For the former object, a new set of problems must be formulated and a start made towards their solution; for the latter, a new method of procedure was practically necessary, for it was almost certain that no one would repeat the calculations which appeared to have been pushed as far as was humanly possible with the adopted methods. These two sides of Hill's work are quite distinct even though they both start from the same memoir.

The older lunar theorists had taken the ellipse as a first approximation, that is, at the start the action of the sun was neglected. Hill proposed a first approximation in which a

portion of the sun's action should be taken into account. If an examination of Delaunay's final expressions for the longitude, latitude, and parallax be made, it is seen that the infinite series proceed along powers of five parameters and that the rate of convergence along powers of one of these, the ratio of the mean motions of the sun and moon, is far more slow than along powers of the others, owing to the presence of large numerical factors. Hill conceived the idea of neglecting all these other parameters and then finding the series in powers of this ratio alone, with all needed accuracy. He set up the equations of motion, solved them and gave formulas of recurrence which enabled him to avoid the slow approximation methods which generally advanced the degree of accuracy by only one or two powers of the ratio at each step; in his method it advances by four powers of this ratio. The expressions are worked out both literally and numerically, the latter being taken to fifteen significant figures, a number not very much in excess of what is actually required.

As obtained, the coordinates are referred to axes which move with the mean velocity of the sun round the earth and in this form the expressions involve the time through its presence in multiples of a single angle. In the transformations which are necessary to convert rectangular coordinates to polars Hill makes full use of the method of " special values " or, as it is now called, of harmonic analysis and synthesis. He was always very fond of this kind of transformation, using it much in later years and even attempting to systematize its use when many hundreds of terms were present.

It would be unjust in this connection not to mention the indebtedness of Hill to Leonard Euler, probably the greatest of lunar theorists since Newton. Euler, as Hill remarks, had had the idea of starting the theory in the same way with moving rectangular axes and with the same first approximation and had carried it out to a considerable extent in his theory published in 1772 and in a later memoir.

The further steps outlined by Euler and quoted by Hill consist of the determination, step by step, of the terms arranged in powers of the parameters which had been neglected. Each step is to consist of the complete calculation with all needed accuracy of the function of the time and the ratio of the mean motions which multiplies each combination of powers of the remaining parameters. There are several difficulties in fol-

lowing this process. The chief one, which Hill solved in the memoir on the perigee of the moon, is the determination of the first new angle containing the time which arises in the second approximation. In later approximations this angle also involves all the parameters and other methods are needed to find the new portions depending on them. Euler possibly foresaw this; Hill certainly did, but he never carried his work to the degree of approximation which would need them. The method has been used by the writer for the construction of a complete theory of the moon's motion.

The expressions for the coordinates, referred to the moving rectangular axes, have another property: they form Fourier series and are therefore periodic. The resulting orbit in this moving plane is consequently closed. Recognizing this fact, Hill draws the curve. But he saw that the orbit was of interest apart from its application to the lunar problem, for he immediately proceeds to trace, with some care, orbits for values of the ratio of the mean motions other than that which holds for the actual moon and sun. He thus obtains a family of such orbits. It is Hill's idea of the periodic orbit which, developed chiefly by Poincaré and G. H. Darwin, has given new life to the whole subject of celestial mechanics and has induced many mathematicians to investigate on these lines. The treatise of the former, Les Méthodes nouvelles de la Mécanique céleste, is based mainly on this idea. Darwin actually traced many such orbits under varying conditions.

There is still another portion of this memoir which has been largely used as a foundation for investigations into the stability of celestial systems. If the eccentricity of the earth's orbit round the sun be neglected, it is possible to write the relative energy equation in a finite form. Referred to the same moving axes, the square of the velocity can, in fact, be expressed as a finite algebraic function of the coordinates. Since the square of the velocity can never be negative, this function, equated to zero, gives the equation to a surface which the moon cannot cross. As the surface consists of various ovals and folds, we can obtain certain limitations on the path of the moon and therefore carry forward the question of the stability of its motion one important step. Hill draws the surfaces for a limited case. Darwin made extensive use of a similar diagram for a more extended case, and many others have followed on the same lines.

Thus this memoir of but fifty quarto pages has become fundamental for the development of celestial mechanics in three different directions. Poincaré's remark that in it we may perceive the germ of all the progress which has been made in celestial mechanics since its publication, is doubtless fully justified. It has sometimes been said that Hill did not appreciate at the time the importance of his work. Hill was far too modest about his own achievements to lay any such stress on his productions as has the scientific world. But it does not require an extended study of his memoirs to see that his vision often went beyond the particular matter in hand.

The second memoir of 1877, "On the part of the motion of the lunar perigee which is a function of the mean motions of the sun and moon," has already been referred to. It is essentially a continuation of that part of the researches which deals directly with the lunar problem, although published a few months earlier. While not so far reaching from the point of view of future developments, it is even more remarkable as an exhibition of Hill's powers of analysis. In it, the determinant with an infinite number of elements is raised from a nebulous possibility to an instrument of computation. Hill's periodic orbit contained only two of the four arbitrary constants which the complete solution of his differential equations requires. He therefore proceeds to find an orbit—no longer periodic—differing slightly from the periodic orbit but still satisfying the differential equations to the first power of the small variation. The equations obtained are two of the second order and linear with respect to the two unknown dependent variables. An able analysis with the use of known integrals, enables him to reduce the solution to that of one of the second order in the normal form

$$\frac{d^2p}{dt^2} + Vp = 0,$$

where V is a known Fourier series depending on the time. Knowing the form of the solution

$$p = \Sigma a_i \cos \{c(t - t_1) + 2i(n - n')(t - t_0)\}$$

from previous work in the lunar theory, a form he justifies by general considerations, Hill substitutes and obtains an infinite series of linear equations for the determination of the

unknowns a_i. But c is also unknown and it does not enter in a linear form. The a_i are eliminated by means of a determinant with an infinite number of rows and columns equated to zero; this is therefore a determinantal equation to find c, the main object of the investigation.

Then follows a remarkable series of operations. The determinant is reduced to a convergent form (though it was left to Poincaré to furnish the proof of convergence) by dividing each row by a suitable factor which reduced every element of the principal diagonal to unity. Next, the unknown c must be isolated; Hill achieves this by recognizing that if c be a root so must $c + 2i$ be also a root and that therefore the roots can be expressed by a cosine function. On the assumption that there are no other roots, he equates the determinant to the cosine function, obtaining the constant by comparing the highest (infinite!) power of c on each side of the equation. A particular value of c (not a root) can be inserted in the identity thus obtained. In this way, Hill reduced the work to the computation of an infinite determinant every element of which is known. He gives a general method for this expansion which enables him to tell at once the order of the terms neglected when the series is cut off at any place. Each term of this series, however, consists of singly, doubly, . . . infinite series which must be summed. The labor at this stage was very great and it caused a corresponding liability to error. Hill carried it through with complete success in its general form, afterwards substituting numbers and determining c to sixteen significant figures. The principal part of the motion of the moon's perigee is immediately deducible from c. President Woodward relates that the determinant was solved during one of two trips which Hill made to the northwest region of Canada; I imagine, however, that this statement refers to the method to be adopted rather than to the actual computation.

The story of these two memoirs is incomplete without a notice of the work of J. C. Adams on somewhat similar lines. Almost immediately after their publication, a brief paper by him appeared in the *Monthly Notices* of the Royal Astronomical Society. He had also taken up Euler's idea and had obtained the variation orbit as a first approximation. But he turned to the motion of the node instead of to that of the perigee. The investigation here follows lines very similar to those of Hill, the solution of the infinite determinant being closely analogous.

It is convenient at this stage to take up Hill's work rather by subject than in chronological order. The periodic orbit used with such excellent results in the lunar theory is tried later (1887) on the motion of the satellite Hyperion as disturbed by Titan and the results applied in a following paper to obtain the mass of the latter. These were written before the publication of Poincaré's researches. Only on one occasion did Hill make it the subject of a theoretical research and it was then probably stimulated by reading Poincaré's Mécanique céleste. As the title, "Illustrations of periodic solutions in the problem of three bodies," indicates, it consists of applications to certain bodies in the solar system.

From time to time a paper was published advancing the applications to the lunar theory. In one, the periodic orbit is extended so as to include the terms which depend on the ratio of the parallaxes of the sun and moon as well as on the ratio of the mean motions. In another paper the terms dependent on the latter ratio and on the first power of the solar eccentricity are computed. In still another paper he calculates the expression for the principal part of the motion of the moon's perigee as far as m^{11} literally in order to settle the correctness of Delaunay's value, which had been questioned as to certain of the earlier powers of m by Andoyer. Beyond these, he seems to have made no effort to continue the work in this direction. Possibly this was due to the heavy labor on the theories of Jupiter and Saturn which engaged him at least until 1892. In fact, as early as 1888 he stated in a letter to Sir George Darwin that he scarcely expected to proceed with the subject.

His fondness for Delaunay's methods has already been mentioned. One of his most valuable memoirs is an application of them to the calculation of the smaller perturbations of the moon's motion which arise from the action of the planets and the figure of the earth. Hill, using Delaunay's methods and results, showed, in a short paper on the Jovian evection, that the whole action of the earth and sun on the moon could be treated as known from the start and that therefore only one approximation was needed in order to get the effect of any disturbance whose square could be neglected. All later investigators have used this method. The formulas of Delaunay are literal, while Hill's final equations for the calculation of the effect of any small disturbing force have the great ad-

vantage of well-determined numerical coefficients to be multiplied by the constants which depend solely on the nature of the given force.

In an earlier paper he had also shown how the disturbing function for direct planetary action can be expressed as a series of products, one factor in each product containing the coordinates of the earth and moon only, while the other contained those of the earth and planet only. The former could therefore be computed once for all; it was the latter which required separate computations for each planet. This paper has also formed the basis for all the complete calculations of the planetary disturbing forces which have been made since its publication in 1883.

But Hill's most extensive application of Delaunay's theory is made in its original form to the calculation of the inequalities produced by the figure of the earth. While he carried these to the degree of accuracy needed for observation, the method appears to be somewhat long and complicated. It has to be applied in a literal form and this requires expansions which converge very slowly. As a matter of fact a few days work with the methods which he adopted for the planetary terms will furnish the inequalities with all needed accuracy. In the first part of this paper Hill, not content with the values for the flattening of the earth which were then in use, deduced one directly from a large number of pendulum observations all over the earth. The result, 1/288, is considerably larger than most of the other determinations and notably so than that of Helmert, 1/298, deduced from the same class of observations. The memoir occupies over a hundred and forty pages and must have demanded an enormous amount of careful and accurate algebraic computation. To complete the account of his work on the lunar theory, mention must be made of his calculation, by de Pontécoulant's method, of the principal inequalities produced by the motion of the ecliptic. Hansen was the only writer who had found the term in longitude as well as in latitude, and nearly all his calculations of the small perturbations are doubtful. Hill, of course, obtained correct results as far as he went in the matter.

Newcomb, who had taken charge of the American *Ephemeris* in 1877, soon induced Hill to undertake the theories of Jupiter and Saturn, and so give material assistance in his plan of forming new tables of the planets. The method

adopted is that of Hansen with only a slight modification which consisted in expressing the computations directly in terms of the time instead of using two auxiliary angles. That he used an old method in preference to devising a new one is perhaps unfortunate even though the result leaves little to be desired. Had he taken more time over the preliminary stages we should probably have had something new and original, for Hill was then at the height of his powers as a mathematician. But he was doubtless under some pressure from Newcomb, who wished to complete his great plan during his tenancy of the office of director, and Hill himself may have desired to finish the calculations as soon as possible in order that he might return to West Nyack. However this may be, he completed the task successfully as may be judged from the small residuals which he obtains after a comparison with observations extending over 150 years. The tables which he formed from the theories of the two planets are now used in most of the national ephemerides.

In 1882 Hill published a memoir of some length on Gauss's method for computing the secular perturbations of the planets. Gauss had outlined only the general idea. Hill takes it up and develops in detail the formulas to be used. In the course of the work he finds that a considerable portion of the calculation depends on three elliptic integrals which may be needed for values of the argument up to 50°. Consequently a large part of the paper consists of the tabulation of these to eight places of decimals at intervals of a tenth of a degree; the first and second differences are also printed so that the tables are in form ready for interpolation. As an example he computed the secular perturbations of Mercury by Venus with great accuracy. Two further papers on the same subject appeared in 1901.

In these years Hill published a number of short papers in the *Analyst*, a journal no longer in existence. Sometimes they are merely solutions of well-known problems, at other times, simplifications of proofs of theorems which had evidently presented difficulties to him and which he felt needed elucidation or elaboration—two favorite words with him. But Hill was not a great expositor: even for those familiar with the subject his work is often difficult and sometimes obscure. Newcomb used to say that if Hill had only the faculty of explaining his own ideas he might have avoided many an

error and saved much time. Hill's ability to assimilate and extend the work of his predecessors, at any rate in his earlier days, doubtless prevented him from appreciating the difficulties of others. When the reader is used to Hill's style of composition and his general plans in writing out what he had to say, his arguments are much more easily grasped, but he is rarely anything else than concise.

In his last years Hill still continued to publish in spite of failing health. He covered a variety of topics, several of them quite away from the region of celestial mechanics. One of the most extensive of his papers is a memoir on dynamic geodesy, the last in the fourth volume of his collected works and not previously published elsewhere. Some later papers on a variety of subjects will appear in a fifth volume, to be published, like the previous four volumes, by the Carnegie Institution of Washington.

If an attempt is made to regard Hill's work as a whole and to try to find out his point of view, one thing stands out clearly: a desire to obtain exact knowledge about natural phenomena, in however limited a field, which could be expressed in a numerical form. He never seemed to hesitate about making long calculations and apparently had a positive liking for obtaining his results to many places of decimals. But unlike the tendencies of those who engage much in computation, his mind did not seem to get cramped by figures. Not only could he see both trees and wood, to adopt a familiar simile, but could trace paths in the wood and keep his eyes open for roads which led in directions other than that he was exploring. He had remarkable ability for algebraic manipulation which reached its highest manifestation in the memoir on the perigee of the moon. The more modern sides of mathematics appealed to him but little; if a formula or a series could be reduced to numbers, such questions as convergence did not trouble him much, a point of view which has later been fully justified by Poincaré. He seemed to take but little color from the work of others. Even when, as in many cases, he starts with the results of some previous investigator his writing shows only slight influence of the source of his ideas; it is individual and carries the reflection and methods of his own mind.

Hill never married. He lived much alone, but while resident in Washington would take long walks on Sunday, often with one or two companions. He was fond of botany without

being a collector of specimens and found his chief outdoor recreation in the study of nature. He made two long canoe trips in the northwest of Canada. A carefully written diary illustrated with photographs of the second expedition which took him by rivers and lakes from Lake Superior to Hudson's Bay, is amongst the books which he left in his will to Columbia University.

He was president of the American Mathematical Society from 1894 to 1896, and served as lecturer on celestial mechanics in Columbia University from 1898 to 1901. The manuscript of his lectures shows that they must have cost him much labor; it contains long algebraic developments and is apparently intended to be a more or less complete account of the methods by which the motions of the moon and planets are calculated. His numerous honors include foreign membership in the Royal Society, the Paris Academy, and the Belgian Academy. He received the Schubert Prize (Petrograd), the Damoiseau Prize (Paris), the Gold Medal of the Royal Astronomical Society and in 1909 the Copley Medal of the Royal Society.

His chief characteristic was a single-minded devotion to the subject which he had made his own. A highly sensitive conscience was always apparent in his dealings with the world: one year he refused to accept the salary of his lectureship at Columbia because no students then appeared to attend the course, and this in spite of the fact that the endowment left him absolutely free to lecture or not as he chose. In later years, he rarely left West Nyack, owing to ill health. He died on April 16, 1914, from heart failure and was buried near the graves of his ancestors not far from his home.

E. W. BROWN.

DICKSON'S LINEAR ALGEBRAS.

Linear Algebras. By L. E. DICKSON, Ph.D. (No. 16, Cambridge Tracts in Mathematics and Mathematical Physics.) Cambridge, University Press, 1914. 8vo. viii + 73 pages.

> And still they read, and still the wonder grew,
> That one small tract contain so much. . . .

A SUBSTANTIAL and systematic introduction to general linear algebras, associative and non-associative, a revision of Cartan's theory of linear associative algebras over the field of

complex numbers, the results of Wedderburn's theory of such algebras over a general field, the relation of linear algebras to finite and infinite groups and bilinear forms, the consideration of various special algebras and a wealth of historical and bibliographical references in footnotes—in seventy-three pages!

The space at the author's disposal was doubtless limited. To have compressed so much material into it is indeed an achievement. With such an end in view the author could not afford to bandy words. He must needs make a point of being brief. And he has made this point well. His sentences are concise, and more often than not are condensed by the use of symbols rather than words or, now and then, by some ingenious form of brevity. The neatest of these is the use of (3_1), (8_2), etc., to denote "the first of the formulas (3)," "the second of the formulas (8)," etc. The equality sign is called upon to do duty in various rôles: thus, to denote the equivalence of two references, "*Proc. London Math. Soc.*, . . . (= *Coll. Math. Papers*, . . .)"; or to indicate the coordinates of a point, "the point $C = (c, d)$."

In attempting to attain to brevity, an author runs the danger of sacrificing clearness. Doubtless it is to avoid this pitfall that some authors spare no space in inserting every last bit of reasoning in their proofs and discussions. But a text-book from the pen of such an author affords the reader merely mental entertainment, whereas, in the reviewer's opinion, the ideal text-book on any except the elementary subjects should also require of the reader mental exercise. And often the more vigorous the exercise required, provided it remains within the possibilities of the reader, the more really lasting and beneficial is the entertainment.

Needless to say, the book under review is of the entertainment-plus-exercise type. The author demands continually of the reader that he supply bits of reasoning. But he is always very careful to make adequate suggestion as to their nature, usually by reference to previous sections and formulas. Thus actual clarity does not suffer; there is not an obscure place in the tract if one reads every word at its full value. Especially is the author to be complimented on his able revision of the complicated theory of Cartan in Part II.

For a complete understanding of a book, a reader needs not only to master the details but, as he reads, to strip them of their unessentials and assign them to their proper place in a skeleton

structure of the whole subject, so that in the end he will see the aims, methods and facts, their significance and relative value, in a true perspective. Naturally he must do most of this digestive work himself, but the author may furnish him invaluable help by appropriate comments now and again. To do this, and do it well, is no easy task at the best. Still it is simpler, certainly, in a book of the type under review, in which the abstraction from the unessentials is already partly done, than in one of the purely entertainment type, in which such abstraction is rendered exceedingly difficult by the great mass of detail. But the present author makes no use of this advantage; in fact he makes next to no attempt to keep the reader properly oriented. As a result, his book presents always one and the same aspect, a desert of statement and proof with not a refreshing oasis in sight, where the reader may pause to rest and take account of stock. An amount of space, equivalent, say, to a half dozen unbroken pages, devoted to "oases" distributed judiciously throughout the book, would, in the reviewer's opinion, improve it greatly in raising the quality of clearness of the whole up to the standard of that attained in the details.

The tract is divided into four parts. Part I contains "definitions, concrete illustrations and important theorems capable of brief and elementary proof." It deals with the general linear algebra, associative and non-associative. Now the associative algebras of real interest are those possessing principal units, for in such an algebra without a principal unit division, if possible at all, is not unique. But with the introduction of non-associative algebras into the discussion, the presence or absence of a principal unit takes on new meaning. Hence in reading this first part, the beginner should keep an eye peeled for the appearance and disappearance not only of "associative," but also of "principal unit." In Part II we find the heart of the general theory, presented from Cartan's point of view for linear associative algebras over the field of complex numbers. The author chooses Cartan's treatment rather than those of Molien, Wedderburn, or Frobenius, because it alone remains within the bounds of linear algebras for its methods and proofs. The other treatments depend in whole or in part on the theory of groups or bilinear forms, and it is the purpose of the writer not to appeal to these to establish his results. The relations of these allied the-

ories to linear algebras are adequately discussed, however, in Part III. Part IV contains the statement and elucidation of Wedderburn's results concerning linear algebras over an arbitrary number field. Numerous examples, illustrative of the theory, are given in smaller print throughout the book.

Part I begins with the development of the system of ordinary complex numbers. We are disappointed to find that the method of procedure here does not wholly conform to that used later in developing the system of general hypercomplex numbers. In this latter method the n-tuple form $(x_1, x_2, \cdots, x_n)$ of the hypercomplex number x is discarded just as soon as the processes, addition and multiplication by a scalar, requisite to obtain the more useful form $x = e_1x_1 + e_2x_2 + \cdots + e_nx_n$, where $e_1, e_2, \cdots, e_n$ are the n units, have been defined. But in the development of the ordinary complex numbers (and also in that of the matric algebra which follows in §§ 3, 4) there is no explicit mention made of multiplication by a scalar, and the couple form (a, b) of the complex number is retained almost to the end.

In § 2 a set of complex numbers is defined as a number field "if the rational operations can always be performed unambiguously within the set." The reviewer has racked his brain in a vain attempt to conceive a reason for the insertion of "unambiguously." If a rational operation on ordinary complex numbers—the definition can refer only to these, since no others have as yet been defined—is performable at all, i. e., if it is not division by zero, then the result is certainly unique.

The author adheres rigorously to the use of the concept of number field throughout. Thus, he defines (§ 5) as a linear algebra* over the field F the set of all numbers $\sum_{i=1}^{n} x_ie_i$, with coordinates x in F, subject to combination by suitably defined addition, subtraction, multiplication by a scalar in F, and by a distributive multiplication with the multiplication table $e_ie_j = \sum_{k=1}^{n} \gamma_{ijk}e_k$ $(i, j = 1, \cdots, n;\ \gamma\text{'s in } F)$.

In such an algebra, right-hand division, i. e., solution of $xx' = y$ $(y \neq 0)$ for x', is possible and unique if and only if the right-hand determinant of $x, \Delta(x) = |\sum_{i=1}^{n} x_i\gamma_{ijk}|$ $(j, k = 1, \cdots,$

* From now on in this review "algebra" will be used in the sense of "linear algebra."

n), is not zero. The left-hand determinant $\Delta'(x)$ is similarly related to left-hand division (§ 6). The number ϵ, if it exists such that $x\epsilon = \epsilon x = x$ for every number x of the algebra, the author sometimes calls "principal unit," but just as often "modulus," a term aptly and justly described by Study as "den auf alles Mögliche angewendeten Ausdruck 'Modul.'"* A necessary condition that there exist a principal unit is that both kinds of division be, in general, possible. The condition is proved sufficient if the algebra is associative, but for a non-associative algebra the author does not enlighten us (§ 7).†

In § 8 linear transformation of the units of an algebra is considered and the equivalence of two algebras under such linear transformation defined; in §§ 9, 10 it is noted that the general number x of an associative algebra is the root of an equation of degree $\leqq n + 1$ and satisfies any algebraic identity in an ordinary complex variable made up of functions without constant terms, or if the algebra moreover contains a principal unit, is a root of an equation of degree $\leqq n$ and satisfies *any* algebraic identity in an ordinary complex variable. By application of these results the author shows, in § 11, that every number of an associative algebra over the field $F(a)$ of real numbers, which has no nilfactors (divisors of zero), is a root of a quadratic equation, and hence obtains a neat proof that the only such algebras are $F(a)$ itself, the field $F(i)$ of ordinary complex numbers, and the algebra of real quaternions. In § 12 we find a discussion of the simplest properties of real quaternions, in § 13 a proof that the complex quaternion algebra and the complex matric algebra with four units are equivalent, though their real sub-algebras are not. In § 14 the author represents Cayley's eight unit algebra, a non-associative generalization of real quaternions, as a "quasi-binary algebra with real quaternion coordinates" and thus gives simple and pretty proofs that this algebra contains no nilfactors and that in it, too, the norm of a product equals the product of the norms.

We now return to the general theory and introduce the right and left-hand characteristic determinants and equations $\delta(\omega)$, $\delta'(\omega)$, $\delta(\omega) = 0$, $\delta'(\omega) = 0$, where $\delta(\omega)$ and $\delta'(\omega)$ are derived from $\Delta(x)$ and $\Delta'(x)$ by subtracting ω from each

* Encyklopädie der math. Wiss., I, A, 4, p. 162.

† It is a simple matter to construct an example to show that the condition is not sufficient for a non-associative algebra.

element of the principal diagonals. The general number of an associative algebra is a root of both $\delta(\omega) = 0$ and $\delta'(\omega) = 0$. Suitable variations of this theorem for an algebra with a principal unit and for an arbitrary algebra are given. In § 17 a function of the x_i and the γ_{ijk} which reproduces itself under an arbitrary linear transformation of units with a factor depending only on the coefficients of the transformation is called a covariant (invariant, if it involves only the γ's). The determinants $\Delta(x)$, $\Delta'(x)$, $\delta(\omega)$, $\delta'(\omega)$ are absolute covariants of the general algebra. The proof given in § 16 of this is incomplete,* since the statement "let $\Delta(x)$ be not identically zero, so that there exists a modulus" assumes the algebra associative and hence leaves the theorem unproved for non-associative algebras for which $\Delta(x) \not\equiv 0$. The theorem is applied in § 18 to obtain the three (two) types of non-equivalent binary algebras over $F(a)[F(i)]$ with a principal unit.

The rank r of an associative algebra is the least positive integer such that x^r is a linear combination of lower powers of x whose coefficients are rational functions of $x_1, \cdots, x_n$ with coefficients in F. The corresponding equation $R(x) = 0$ is called the rank equation. If the algebra has a principal unit, $R(\omega) = 0$, $\delta(\omega) = 0$, $\delta'(\omega) = 0$ all have the same distinct roots. As an application, the author gives Study's classification of complex ternary associative algebras with a principal unit into five non-equivalent types according to the five possibilities for the degree and the multiplicity of the roots of the rank equation (§ 20).

The author defines an associative algebra with n units and a principal unit as reducible in its field F if it contains n numbers $e_1, \cdots, e_p; E_1, \cdots, E_q$, linearly independent with respect to F,

* The author's proof goes astray in assuming the existence of a principal unit. But the only use he makes of this assumption is to show, after he has proved that $D(X) = c\Delta(x)$, where c depends only on the coefficients c_{ij} of the transformation of units, that $c = 1$. Now, by inspection, c is the quotient of a homogeneous expression of degree $2n$ in the c_{ij} by $|c_{ij}|^2$. Since c cannot vanish unless $|c_{ij}|$ vanishes—otherwise we could exhibit values for the x_i, γ_{ijk} for which $\Delta(x) \neq 0$, while $D(X) = 0$—and $|c_{ij}|$ is irreducible, $c = k|c_{ij}|^l$, where k is independent of the c_{ij}. But evidently $l = 0$, and, by applying the identical transformation of units, $k = 1$.

Professor Dickson's proof holds for any algebra with a principal unit. In a letter to the reviewer he points out that the case of an algebra without a principal unit may be treated by enlarging it to an algebra with a principal unit by adjoining a new unit (as in the proof of Theorem 3, § 15), and also calls attention to a later proof by Miss Hazlett, *Annals of Mathematics*, vol. 16 (1914), p. 2.

such that $e_iE_j = E_je_i = 0$, $(i = 1, \cdots, p;\ j = 1, \cdots, q)$, and proves that such an algebra is the direct sum of two algebras.* He then sets up Scheffer's criterion for reducibility. In § 22 the direct product of any two algebras is defined and Wedderburn's generalization of Scheffer's theorem identifying an associative algebra, which has the quaternion algebra as a subalgebra and its principal unit as principal unit, as the direct product of the quaternion algebra and a second algebra.

In the last main paragraph (§ 23) of Part I a set of normalized units for any algebra is developed. Since their main purpose is to form a point of departure for Part II, we have need to describe them here only for an associative algebra with a principal unit ϵ. Suppose that the solutions of $\delta(\omega) = 0$ for a particular number a of the algebra are $\omega_1, \cdots, \omega_h$ of multiplicities $m_1, \cdots, m_h$, and consider one of these solutions, say ω_1. Now $a - \omega_1\epsilon$ is a nilfactor, that is, there exist solutions $y \neq 0$ of $(a - \omega_1\epsilon)y = 0$ or $ay = \omega_1 y$; take a complete system of solutions y, say t in number, as the first t of the new units $e_1, e_2, \cdots$. If $t < m_1$, there exist solutions z of $az = \omega_1 z + \sum_{i=1}^{t} c_i e_i$; take a complete system of solutions z, say t_1 in number, for the next t_1 of the new units. If $t + t_1 < m_1$, repeat; we will finally obtain m_1 numbers $\alpha_1, \alpha_1', \cdots, \alpha_1^{(m_1-1)}$, such that $(a - \omega_1\epsilon)\alpha_1^{(i)}$ is a linear function of the preceding α_1's. Similarly, there exist, corresponding to the root ω_j, m_j numbers $\alpha_j, \alpha_j', \cdots, \alpha_j^{(m_j-1)}$, such that $(a - \omega_j\epsilon)\alpha_j^{(i)}$ is a linear function of the preceding α_j's. The $n = m_1 + m_2 + \cdots + m_h$ new units thus obtained are linearly dependent; they are termed a set of units normalized relatively to the number a.

Part II consists of a "revision of Cartan's general theory of complex linear associative algebras with a modulus." In § 25 units having a character are developed. Denote the principal unit, expressed in terms of the units of § 23, by $\epsilon = \epsilon_1 + \epsilon_2 + \cdots + \epsilon_h$, where ϵ_i is a linear combination of the α_i's. (The ϵ_i are called the partial moduli.) Then the number η is said to have the character (j, k) if $\epsilon_j\eta = \eta$, $\eta\epsilon_k = \eta$, and every other combination of η with an ϵ_i by multiplication is zero. Now $\epsilon_j\alpha_j^{(i)} = \alpha_j^{(i)}$, $\epsilon_k\alpha_j^{(i)} = 0$ and in particular $\epsilon_i^2 = \epsilon_i$,

* He might have generalized the definition by demanding, instead of the existence of a principal unit, the existence of two numbers e, E, where e (E) is a linear function of the e_i(E_i) with coefficients in F, which is not a nilfactor with respect to all other such linear functions, and the proof would still hold with but slight variation.

$\epsilon_i\epsilon_j = 0$; furthermore there exist m_j linearly independent linear combinations of the $\alpha_j^{(\omega)}$, such that the product of any one of them into a certain ϵ_i gives itself, and its product into any other ϵ_i gives zero, i. e., such that they have definite characters $(i, \cdot)$. Hence we have obtained n new units $\epsilon_1, \epsilon_2, \cdots, \epsilon_h, \eta_1, \cdots, \eta_k$ each having a definite character. We wish further to show that we can choose the η's so that each is nilpotent, i. e., so that some power of it is zero. Now the square of every η of character (i, j), $i \neq j$, is zero, since the product of a number of character (i, j) by a number of character (k, l) is zero, if $j \neq k$. And since the sum or product of two numbers of like character, if not zero, is of that character, all the numbers of character (i, i) of the algebra form a sub-algebra Σ_i; it is possible to choose the units of this sub-algebra, other than ϵ_i, as nilpotent numbers. Thus we have arrived at a set of normalized units consisting of the partial moduli and $n - h$ nilpotent numbers each having a definite character. We shall refer to such a set of normalized units as being of type K.

We now separate algebras into two categories, according as the determinant $\Delta(x)$ does not or does contain a non-linear irreducible factor. To prove the principal theorem for algebras A_1 of the first category, i. e., that for such an algebra a set of units of type K can be found having the further property that $\eta_i\eta_j$ is a linear function of those η's whose subscripts exceed i and j and have the same character as $\eta_i\eta_j$ (property L), the author shows that the nilpotent numbers of A_1 form a sub-algebra N, for which units η of definite characters enjoying the property L can be found. Conversely, if an algebra is given possessing a set of units of type K enjoying the property L, the γ_{ijk} of the multiplication table are determined (except for the non-vanishing ones in the $\eta_i\eta_j$) and inspection shows that $\delta(\omega) = (x_1 - \omega)^{k_1} \cdots (x_h - \omega)^{k_h}$, that is, that the algebra is of the first category. An algebra such as N is called nilpotent.

To every irreducible factor of the characteristic determinant for an algebra A_2 of the second category, corresponds a sub-algebra of A_2, just as in the case of an algebra A_1. But here at least one of these irreducible factors is not linear. Each sub-algebra corresponding to such an irreducible factor contains linear combinations of nilpotent numbers which are not nilpotent. These numbers are peculiar to algebras A_2

and form the basis of Cartan's treatment of them. The details of the treatment are too intricate to permit of an adequate description in a brief space. We content ourselves with a statement of the principal results. First, any algebra A_2 contains a sub-algebra equivalent to a matric algebra with more than one unit. Hence Cartan's classification of algebras into those of the first and second categories coincides with that of Scheffers into those without and with quaternion sub-algebras. As the final result we have a process by means of which the algebras A_2 may be derived from the algebras A_1: the general number of an A_2 is obtained from the general number of an A_1, expressed in terms of units of type K having the property L, " by regarding the coefficient of ϵ_i to be a square matrix of p_i^2 elements and that of η_l, of character (i, j), to be a rectangular matrix of p_i rows and p_j columns, these matrices to be regarded as commutative with each ϵ and η." Thus we obtain from the units of an A_1, the multiplication table and the characteristic determinant corresponding to them, the units, multiplication table and characteristic determinant of an A_2.

In §§ 49, 50 the composition of an algebra with respect to the presence or absence of invariant sub-algebras is discussed. A sub-algebra I of an algebra A is termed invariant if the product of any number of I and any number of A in either order is a number of I. An algebra having no invariant sub-algebra is called simple and one having no nilpotent invariant sub-algebra semi-simple. A general algebra, A_1 or A_2, is the sum, but not necessarily the direct sum, of a semi-simple algebra and a nilpotent invariant sub-algebra. A semi-simple algebra, if not simple, is the direct sum of simple algebras and conversely. Finally, a simple algebra is matric, unary if an A_1, non-unary if an A_2, and conversely.

In concluding Part II the author shows that a commutative algebra must be an A_1 and, in particular, an A_1 which is the direct sum of its sub-algebras $\Sigma_1, \cdots, \Sigma_h$.

Part III begins with a consideration of the correspondence between associative algebras and linear groups. To each number y of any algebra correspond two linear homogeneous transformations (y), $[y]$ of the variables $x_1, \cdots, x_n$ into the variables $x_1', \cdots, x_n'$, defined by the equations obtained from the products $x' = xy$, $x' = yx$. If the algebra is associative, the set of transformations (y) is closed and so is the set of

transformations $[y]$. If further the algebra contains a principal unit, there corresponds to it in each set the identical transformation, and hence the inverse of any transformation corresponding to a non-nilfactor exists (and belongs to the same set). Thus the two sets of transformations (y), $[y]$, corresponding to the non-nilfactors y, form groups G, G'. These groups are simply transitive, mutually commutative (reciprocal in the sense of Scheffers), and each is its own parameter group. Conversely, any two simply transitive reciprocal groups of linear transformations can be transformed by the same change of variables into a pair of groups G, G', defined by an associative algebra with a principal unit. A classical example of the representation of linear transformations by hypercomplex numbers is then given: if q and Q are variable and q_1, q_2 given quaternions, then $Q = q_1\, q\, q_2$ represents the 7-parameter group of transformations of similitude in R_4 leaving the origin invariant; the group represented by the equation when, in particular, q_1 and q_2 are conjugate and q and Q vectorial quaternions, is not the corresponding group in R_3, as the author states, since it does not contain the stretchings from the origin but only the rotations about it.

Section 53 shows that the theory of bilinear forms is equivalent to that of matrices and also, in the last analysis, to that of associative algebras; section 54 explains the relations between finite groups and their group matrices to linear algebras; and section 55 treats of Dedekind's idea of considering the relations giving the values of the products $e_i e_j$ $(i, j = 1, \cdots, n)$ of the units of a commutative associative algebra as ordinary algebraic equations in the units as unknowns.

The decomposition of a linear associative algebra over a field F within this field is precisely the same as that given above (§ 50) for a complex associative algebra within the field $F(i)$, except that instead of the last statement we now have: any simple algebra over F is the direct product of a division algebra and a simple matric algebra, each over F, and conversely. A division algebra is one without nilfactors. The relation of the particular case, $F = F(i)$, to the general one is made clear by the fact that the only division algebra over $F(i)$ is $F(i)$ itself. Further, if F is the field $F(a)$ of all reals, the only division algebras, besides $F(a)$, are $F(i)$ and the algebra of real quaternions. Now, since it can be shown that a commutative matric algebra has a single unit, it follows that

any real commutative algebra without nilpotent numbers is the direct sum of unary algebras equivalent to $F(a)$ and binary algebras equivalent to $F(i)$. Weierstrass showed that a real commutative associative algebra, possessing a principal unit and such that the only algebraic equations with an infinitude of the numbers of the algebra as roots are those in which the coefficients are multiples of one nilfactor, is equivalent within $F(a)$ to just such a sum of real unary and binary algebras and conversely.

In §§ 58–60 are given examples of various types of division algebras previously published by the author. In § 61 results concerning difference algebras, the composition, difference and reduction series of an algebra are given, and, in particular, Wedderburn's theorem to the effect that an associative algebra over F can be decomposed in one and but one way into the direct sum of irreducible algebras with principal units and, if the given algebra has not a principal unit, an algebra without a principal unit. The closing section (§ 62) of the tract contains brief mention of the work of Berloty, Scheffers, and Hausdorff on analytic functions of hypercomplex numbers.

Attention is called to the following misprints: page 4, line 8, read *of* instead of *by*; page 24, last line, $\xi^2-2L\xi$ instead of $\xi^2+2L\xi$; page 32, last line, e_5 instead of e; § 26, last line, $e_3\epsilon_1$ instead of $e_3\epsilon_4$; § 33, third line, $-\omega^5$ instead of ω^5; § 33, fifth line, omit "the" before "normalized units"; page 43, "(71)" applies to both equations; page 48, ninth line from bottom, insert "variables" before [2, 2]. In § 6, the condition $y \neq 0$ is omitted; on page 41, in the statement of the lemma, $u \neq 0$, and in § 57, $k \neq 0$. In light of the complexity of the subscript notation needed, it is surprising that there are so few typographical errors. As the author says, in closing the introduction, "the quality of the printing speaks for itself." There is no index, but the table of contents is so detailed that it practically takes the place of one.

We have already pointed out the lack of general comment; in this connection we wish now to note the meagreness of the treatment accorded to nilfactors. We should like to have seen such an important concept introduced at the first opportunity, namely in § 4, by calling attention to the fact that the algebras given as examples differ essentially, in that the matric algebra contains nilfactors whereas $F(i)$ does not. And it would be natural for us to introduce the definition of nil-

factors in § 6 in connection with $\Delta(x)$ and $\Delta'(x)$, and point out that they are simply the numbers for which these determinants vanish. As a matter of fact, nilfactors are not defined until § 19 and then only for an associative algebra with a principal unit ϵ, whereas the concept may be extended to any algebra; we learn there that $x - \omega\epsilon$ is a nilfactor if and only if ω is a root of the rank equation. This is the first and last mention of " nilfactor " in Parts I, II. It is not even pointed out that the nilpotent number is a special type of nilfactor. We learn that a necessary and sufficient condition that a number be nilpotent is that every root of its characteristic equation be zero, but nowhere are we told that the corresponding condition for a nilfactor is that one root of its characteristic equation vanish.

The conditions on the γ_{ijk}, that multiplication be associative, all but escape mention; we find them buried in a proof on page 59. If the author was to accord them a place at all—as he certainly should—should it not be their natural one in § 5, where multiplication is defined and discussed? Finally, the following points did escape mention: an associative algebra with n units with (without) a principal unit, which is of rank $n(n+1)$, is commutative; a binary algebra with a principal unit is associative and commutative (essential to the complete understanding of § 18).

Though we have had differences with the author, we are in the main at one with him. He has written an able and comprehensive book on an abstruse subject, and a book which satisfies a long felt need, as an introduction and also as an up-to-date book of reference. It should prove invaluable to the beginner and advanced student alike.

W. C. GRAUSTEIN.

SHORTER NOTICES.

Wissenschaft und Methode. Par H. POINCARÉ. Autorisierte Deutsche Ausgabe mit erläuternden Anmerkungen von F. und L. Lindemann. Leipzig und Berlin, Teubner, 1914. vi+283 pp. 5 Marks.

THIS is a straightforward translation of the Science et Méthode of Poincaré, and although the original has never been reviewed in the BULLETIN, it is scarcely necessary at this late date to do so, since every one is familiar with it. In case one were not, he needs only to be informed that the book is the third in the famous series of Poincaré on Science and Hypothesis, the Value of Science, Science and Method, and Last Thoughts. The chief principle of this work is the substantial identity of the methods of science and those of mathematics. The remarkable address on mathematical invention occurs here, in which Poincaré states his views of how mathematical creation takes place, illustrating his remarks from his own experience. In brief the explanation is that after a long and severe study of the nature of a problem, one need not keep on hammering away indefinitely, but should remember that if the right idea for the solution ever comes, it will flash out suddenly and at even inopportune moments, often many days, or weeks, or even years, after the first consideration of the problem. He explains the attractiveness of the right idea when it comes before the mind, by ascribing to our esthetic sense the power of arresting and holding such ideas as fit in with the harmony of the intellectual world we build. It then becomes the function of our logical faculty to verify formally the truth of what has presented itself to us in this way.

The same general notions underlie his statement of the method of the scientist whether in the selection of his facts or in the general development of his theories. The first part of the book discusses from this standpoint the scientist and his work. The second part discusses mathematical educational method. The third part discusses the mechanics of the relativity theory. The fourth part consists of two chapters: in one, a study of the Milky Way by the kinetic theory of gases; in the other, a description of the precise methods of a geodetic survey. Each of these parts contains the well-known ideas of the author in various places. Each part has the stimulating

character of all of the philosophical work of the eminent mathematician, physicist and philosopher. It is a rare occurrence for one so profound in each of these lines to be also so skilled in presenting as clear an exposition of the working of the intellect. The best criticism we can make on the book is that no one can afford to be without it or the original.

JAMES BYRNIE SHAW.

A Text Book of Mechanics. By LOUIS A. MARTIN, JR. Vol. 5, *Hydraulics.* New York, John Wiley and Sons, 1914. 12 mo. 223 pp.

THIS is the concluding volume of a series, the earlier volumes of which (Statics, Kinetics, Mechanics of Materials, Applied Statics) have been previously reviewed in this BULLETIN.*

Approximately 50 pages are devoted to the discussion of liquids at rest; the force exerted by a liquid upon a plane or curved surface; methods of measuring pressure and locating the centre of pressure; applications to gates, locks and dams; floating bodies and the conditions for stability. The remainder of the book treats of liquids in motion, starting with an elementary discussion of the free surface of liquids moving with acceleration. Then comes Bernouilli's classic theorem on the flow of liquids, followed by a treatment of flow through orifices, various formulas for weirs of several types, flow through pipes with applications to hydraulic transmission of energy and calculation of losses in pipe lines, force exerted by a moving liquid upon pipes and on stationary deflecting surfaces, axial or radial flow, impulse wheels and turbines, and their design. There is a good set of problems for a final review.

As in the earlier volumes, the exercises distributed through the text play a vital part in the course, giving students a chance to work out parts of the subject for themselves. These exercises are not too difficult, having been introduced so carefully as to fall within the power of average students. We should add that throughout the series of texts the author presupposes practically no knowledge of physics, stating carefully the basic principles which are to be employed.

The mathematical equipment assumed, however, steadily increases through the series, so that the first volume may be studied before the first course in calculus, while this last volume assumes a very fair knowledge of integral calculus.

* Vol. 16 (1909), pp. 144–7; vol. 21 (1914), pp. 140–3.

The present volume, like those which preceded it, gives every indication of representing an actual course, skillfully taught. The series of volumes afford a most satisfactory introductory course in theoretical and applied mechanics.

F. L. GRIFFIN.

NOTES.

THE closing (June) number of volume 16 of the *Annals of Mathematics* contains the following papers: "Projective classification of cubic surfaces modulo 2," by L. E. DICKSON; "Normal forms for one- and two-sided surfaces," by J. W. ALEXANDER, II; "Minima of double integrals with respect to one-sided variations," by C. A. FISCHER; "A new proof of Sylow's theorem," by G. A. MILLER; "Non-linear integral equations of the Volterra type," by H. GALAJIKIAN; "On some general theorems concerning ordinary closed curves," by ARNOLD EMCH.

PROFESSOR ARTHUR RANUM, of Cornell University, has been appointed an associate editor of the *Transactions of the American Mathematical Society*.

AT the meeting of the London mathematical society held on April 22 the following papers were read: "Note on Dirichlet's division problem," by G. H. HARDY; "Note on a new form of closed linkage," by R. L. HIPPISLEY; "Division of the lemniscate into seven equal parts," by G. B. MATTHEWS; "The influence of the oceanic waters on the law of variation of latitude," by J. LARMOR.

AT the meeting of the Edinburgh mathematical society on May 14, the following papers were read: "On the roots of a derivative of a rational function," by L. R. FORD; "Study of the life and writings of Colin Maclaurin," by C. TWEEDIE; "Recurrence formulas for the functions which represent solutions of the differential equation $d^2u/dx^2 - a^2u = p(p+1)u/x^2$," by H. T. FLINT; "Two simple nomograms," by E. T. WHITTAKER.

THE eighty-fifth annual meeting of the British association for the advancement of science will be held at Manchester, September 7–11, under the presidency of Professor A. SCHUSTER; Sir F. D. DYSON is chairman of section A, mathematical and physical sciences.

THE following courses in mathematics are announced for the present summer semester:

UNIVERSITY OF BERLIN.—By Professor H. A. SCHWARZ: Integral calculus, with exercises, six hours; Applications of elliptic functions, four hours; Elementary geometric treatment of certain problems in maxima and minima, one hour; Colloquium, two hours; Seminar, two hours.—By Professor G. FROBENIUS: Analytic geometry, four hours; Seminar, two hours.—By Professor F. SCHOTTKY: Theory of ellliptic functions, four hours; Plane differential geometry, two hours; Seminar, two hours.—By Professor J. KNOBLAUCH: Partial differential equations, four hours; Theory and application of determinants, four hours; Selected chapters from the theory of surfaces, one hour.—By Dr. A. BYK: Non-euclidean geometry and mechanics, one hour.—By Dr. B. GEHRCKE: Elements of higher mathematics, two hours.—By Dr. C. KNOPP: Ordinary differential equations, four hours; Theory of functions, four hours.—By Dr. D. ROTHE: Differential calculus, with exercises, five hours.

UNIVERSITY OF BONN.—By Professor E. STUDY: Differential geometry, four hours; Seminar, two hours.—By Professor F. LONDON: Elements of differential and integral calculus, four hours; Axonometry and perspective, two hours.—By Professor I. SCHUR: Algebraic equations, four hours; Theory of sets, two hours; Seminar, two hours.—By Dr. J. O. MÜLLER: Selected chapters of integral calculus, one hour; Introduction to differential equations, three hours; Seminar, two hours.

UNIVERSITY OF FRANKFORT.—By Professor A. SCHOENFLIES: Differential calculus, with exercises, five hours.—By Professor L. BIEBERBACH: Algebra, four hours; Algebraic curves, two hours; Seminar, two hours.—By Professor E. HELLINGER: Theory of functions, five hours; Seminar, two hours.—By Dr. O. SZÁSZ: Analytic geometry, with exercises, five hours.—By Professor E. BRENDEL: Spherical astronomy, two hours; Mathematics of insurance, with exercises, four hours.

UNIVERSITY OF GÖTTINGEN.—By Professor D. HILBERT: Calculus of variations, four hours; Selected chapters on the structure of the plane, two hours; Seminar, two hours.—By Professor E. LANDAU: Applications of the calculus and the

theory of functions to the geometry of numbers, six hours; Seminar, two hours.—By Professor C. CARATHÉODORY: Theory of functions, four hours; Partial differential equations of the first order, two hours; Seminar, two hours.—By Professor C. RUNGE: Calculus, with exercises, six hours; Seminar, two hours.—By Professor L. PRANDTL: Elementary mechanics, four hours; Seminar, two hours.—By Dr. H. v. SANDEN: Numerical calculations, with exercises, six hours.—By Dr. E. HECKE: Analytic geometry, with exercises, five hours.—By Dr. L. COURANT: Partial differential equations of mathematical physics, four hours; Selected chapters of the theory of functions, two hours.—By Dr. F. BERNSTEIN: Analytic foundation of the theory of probability, two hours; Mathematics of insurance, two hours; Seminar, two hours.

UNIVERSITY OF LEIPZIG.—By Professor O. HÖLDER: Higher algebra, four hours; Applications of the theory of elliptic functions, three hours; Seminar, two hours.—By Professor K. ROHN: Plane analytic geometry, with exercises, five hours; Descriptive geometry, with exercises, four hours; Determinants, two hours.—By Professor G. HERGLOTZ: Theory of numbers, four hours; Definite integrals and Fourier series, two hours; Seminar, two hours.—By Dr. K. BLASCHKE: Ordinary differential equations, with exercises, six hours; Algebraic analysis, two hours.

UNIVERSITY OF MUNICH.—By Professor F. LINDEMANN: Analytic geometry of space, five hours; Theory of elliptic functions, five hours; Seminar, two hours.—By Professor A. VOSS: Analytic mechanics, four hours; Introduction to hydrodynamics, three hours; Seminar, two hours.—By Professor A. PRINGSHEIM: Higher algebra, II, four hours.—By Professor H. BRUNN: Elements of higher mathematics, three hours.—By Professor G. HARTOGS: Synthetic geometry, four hours.—By Dr. F. BÖHM: Theory of probabilities, four hours; Seminar, two hours.—By Dr. H. DINGLER: Elementary mathematics, four hours; Mathematical treatment of observational data, two hours.—By Dr. A. ROSENTHAL: Differential calculus, with exercises, five hours.

PROFESSORS L. PRANDTL and E. LANDAU, of the University of Göttingen, have been elected to membership in the Göttingen academy of sciences.

THE Helmholtz medal of the Berlin academy of sciences has been awarded to Professor M. PLANCK.

To the list of anniversaries of German professors of mathematics chronicled in the May BULLETIN should be added the celebration by Professor F. MERTENS, of Vienna, of his golden doctor jubilee on November 7, 1914, and his seventy-fifth birthday on March 20, 1915, and by Professor GEORG CANTOR, of the University of Halle, of his seventieth birthday on March 3, 1915.

THE College entrance examination board has now six examiners and twenty-five readers in mathematics, representing twenty-two different schools, colleges, and universities. Professor J. W. YOUNG is chief examiner, and Professor VIRGIL SNYDER chief reader in geometry; Professor F. S. WOODS chief examiner, and Professor C. R. MACINNES chief reader in algebra. Harvard, Princeton, and Yale Universities will discontinue their own entrance examinations after the present year, and will rely entirely on the board examinations.

PROFESSOR A. D. PITCHER, of Dartmouth College, has been appointed professor and head of the department of mathematics in Adelbert College of Western Reserve University. Dr. J. M. STETSON, of the University of Alberta, has been appointed instructor in mathematics at Adelbert College.

PROFESSOR W. M. SMITH, of the University of Oregon, has been appointed associate professor of mathematics at Lafayette College, as successor to the late Professor J. J. HARDY.

PROFESSOR N. F. DAVIS, of Brown University, retires at the close of the present academic year, after more than forty years' active service.

AT the University of Kansas Professor C. A. ASHTON has been promoted from an associate to a full professorship of mathematics, and Professor U. G. MITCHELL from an assistant to an associate professorship of mathematics. Messrs. K. J. HOLZINGER, A. W. LARSEN, and L. L. STEIMLEY have been appointed instructors in mathematics. Mr. H. H. CONWELL has resigned his instructorship to accept an assistant professorship of mathematics in the University of Idaho.

AT Wellesley College Professor HELEN A. MERRILL, at present on leave of absence, has been promoted to a full professorship of mathematics.

DR. H. W. REDDICK, of Columbia University, has been appointed professor and head of the department of mathematics in the Cooper Union, New York City.

PROFESSOR R. D. CARMICHAEL, of Indiana University, has been appointed assistant professor of mathematics in the University of Illinois.

BY mutual arrangement between the departments concerned, Mr. C. H. YEATON, recently appointed instructor in mathematics at Dartmouth College, has resigned to accept a similar appointment at Northwestern University, and Mr. C. R. DINES, at present instructor at Northwestern University, has accepted an instructorship in mathematics at Dartmouth College for the coming academic year.

AT Cornell University Drs. C. F. CRAIG and F. W. OWENS have been promoted to assistant professorships of mathematics.

AT the Massachusetts institute of technology Dr. B. B. LIBBY and Mr. GEORGE RUTLEDGE have been appointed instructors in mathematics.

DR. G. M. CONWELL, of Yale University, has been appointed instructor in mathematics in the New York state college for teachers.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AMOROSO (F.). Complementi di analisi algebrica elementare, con appendice sulle sezioni coniche. 2a edizione, riveduta e migliorata. Napoli, Pirro, 1912. 16mo. 34+229 pp.

BRENKEN (E.). Die Erzeugung der Kurven konstanten Gauss'schen Krümmungsmasses auf Flächen zweiten Grades durch elliptische Zylinder. Papenburg, 1914. 17 pp. M. 1.50

COUTURAT (L.). L'algèbre de la logique. 2e édition. (Collection Scientia.) Paris, Gauthier-Villars, 1914. 8vo. 100 pp. Fr. 2.00

DAVISON (C.). Subjects for mathematical essays. London, Macmillan, 1914. 8vo. 3s. 6d.

D'ENNO (J. G. A.). Ein ganz elementares Verfahren zur Lösung des grossen Fermat'schen Satzes wie auch zur vollständigen Erweiterung und Verallgemeinerung des berühmten Theorems mit Uebungen und Anwendungen auf die Geometrie für die höheren Klassen der Mittelschulen. Trient, G. B. Monauni, 1912. Gr. 8vo. 23 pp.

——. Ein neues Verfahren zur Lösung des grossen Fermat'schen Satzes. Trient, G. B. Monauni, 1912. Gr. 8vo. 15 pp.

DÖLP (H.). Grundzüge und Aufgaben der Differential- und Integralrechnung nebst den Resultaten, neu bearbeitet von E. Netto. 13te Auflage. Giessen, A. Töpelmann, 1912. 8vo. 3+216 pp.

GRODSKIJ (G. D.). Integral calculus. Part 1: Integration of functions. 3d edition. (In Russian.) St. Petersburg, 1912. 8vo. 1+165 pp.

HEINRICHS (G.). Uebungsbuch der Analysis der Infinitesimalen. Herausgegeben von A. Suworin. St. Petersburg, 1912. 8vo. 2+136 pp.

MEHLER (F. G.). Hauptsätze der Elementar-Mathematik zum Gebrauche an höheren Lehranstalten. Bearbeitet von A. Schulte-Tigges. Ausgabe B, Oberstufe. 3ter Teil: Grundzüge und Anwendung der Differentialrechnung. 2te Auflage. Berlin, Reimer, 1912. 7+84 pp.

MUSOTTER (R.). Eine Skizze der Geschichte der Infinitesimalrechnung. Progr. Wien, 1912. 8vo. 21 pp.

NETTO (E.). See DÖLP (H.).

NIELSEN (N.). Elementar Talteori. Kopenhagen, Gyldendal, 1912. 8vo. 144 pp.

ROSTSCHIN (P.). Textbook of the differential and integral calculus. Part 1: Differential calculus; Part 2: Integral calculus. (In Russian.) St. Petersburg, 1911–1912. 4+730+2+508 pp.

SCHULTE-TIGGES (A.). See MEHLER (F. G.).

SUWORIN (A.). See HEINRICHS (G.).

VILLANI (N.). L'equazione di Fermat $x^n + y^n = z^n$ con dimostrazione generale. Lanciano, Masciangelo, 1912. 8vo. 21 pp.

WELMIN (W. P.). On the theory of remainders of the eighth degree in the algebraic number fields. (In Russian.) Warsaw, 1912. 8vo. 21+229 pp.

II. ELEMENTARY MATHEMATICS.

AMMERMAN (C.). See FORD (W. B.).

BERTRAND (G.). Trattato di algebra elementare. Prima traduzione italiana con note ed aggiunti di E. Betti. Nuova edizione per cura di A. Socci. 15a impressione. Firenze, Le Monnier, 1912. 16mo. 7 + 531 pp.

BETTI (E.). See BERTRAND (G.).

BRABANT (F. G.). See MARCON (C. A.).

CAMMAN (P.) et FASSBINDER (L.). Algèbre et géométrie. Classe de 3e A. Paris, Gigord, 1912. 18mo. 87 pp.

CHOPE (R. H.). Junior arithmetic (with answers). London, University Tutorial Press, 1912. 8vo. 8+393 pp.

DEKKER (P.). See NIEMÖLLER (F.).

DOLGUSCHIN (P. A.). Approximations. For gymnasia, Realschulen, trade and technical schools. Revised by the author. 2d edition. (In Russian.) Kieff, 1912. 8vo. 46 pp.

FASSBINDER (L.). See CAMMAN (P.).

FISHER (G. E.) and SCHWATT (I.). Quadratics and beyond. Reprint. New York, Macmillan, 1914. 12mo. 287 pp. $0.90

——. Complete secondary algebra. 2d reprint. New York, Macmillan, 1912. 12mo. 504 pp. $1.35

FORD (W. B.) and AMMERMAN (C.). Plane and solid geometry, edited by E. R. Hedrick. Reprinted with corrections. New York, Macmillan, 1914. 12mo. 321 pp. $1.25

FRENZEL (C.). See MEHLER (G.).

FRIEDMANN (W. G.). Textbook of theoretical arithmetic. For upper classes of the boys' and girls' gymnasia and Realschulen and for private study. (In Russian.) Moscow, 1912. 8vo. 138 pp.

HEDRICK (E. R.). See FORD (W. B.).

JACOB (J.). Lehrbuch der Arithmetik für Mädchenlyzeen und verwandte Anstalten. 1ter Teil: Lehrstoff der 1ten und 2ten Klasse. Wien, F. Deuticke, 1912. 8vo. 3+110 pp.

——. Manuale d'aritmetica per la prima classe delle scuole medie. Versione di R. Marussig. Triest, M. Quidde, 1912. 8vo. 3+64 pp.

JACOB (J.) und SCHIFFNER (F.). Lehrbuch der Arithmetik und Geometrie für Realschulen. Wien, F. Deuticke, 1912. 8vo. 64 pp.

JACOB (J.), SCHIFFNER (F.) und TRAVNIČEK (J.). Arithmetik und Geometrie für Gymnasien und Realgymnasien. Analytische Geometrie der Ebene bearbeitet von J. Travniček. Wien, F. Deuticke, 1912. 8vo. 4+115 pp.

MARCON (C. A.) and BRABANT (F. G.). Responsions papers in stated subjects (exclusive of books), 1906–1911. With answers to mathematical questions and introduction to mathematics, grammar, . . . Oxford, Clarendon Press, 1912. 160 pp.

MARINO (A.). Applicazioni algebraiche alla geometria piana e solida. Milano, 1914. 12mo. 32 pp.

MARKOWITSCH (B. A.). Elementary logarithms. (Elements of the theory of logarithms and of the practice in logarithmic computation.) (In Russian.) St. Petersburg, 1912. 8vo. 8+72 pp. The same with 4-place logarithm tables and samples of other logarithm tables. 8vo. 16+104 pp.

MARUSSIG (R.). See JACOB (J.).

MEHLER (F. G.). Hauptsätze der Elementar-Mathematik zum Gebrauche an höheren Lehranstalten. Bearbeitet von A. Schulte-Tigges. Ausgabe B, Oberstufe. 2ter Teil: Arithmetik mit Einschluss der niederen Analysis, Trigonometrie und Stereometrie. Unter Mitwirkung von C. Frenzel bearbeitet. 2te unveränderte Auflage. Berlin, Reimer, 1912. 8vo. 8+169 pp.

MÜLLER (O.). Tavole di logaritmi con cinque decimali. 12a edizione, aumentata della tavole dei logaritmi d'addizione e sottrazione, per cura di Michele Rajna. Milano, Hoepli, 1915. 24mo. 36+191 pp. L. 1.50

NIEMÖLLER (F.) und DEKKER (P.). Arithmetisches und algebraisches Unterrichtsbuch. In 4 Heften. 1tes und 2tes Heft. Breslau, F. Hirt, 1912. 92+116 pp.

OPENSHAW (P. A.). Public school examination papers in mathematics. With answers. London, Bell, 1912. 135 pp.

PENDLEBURY (C.). A preparatory arithmetic. London, Bell, 1912. 14+185+30 pp.

RAJNA (M.). See MÜLLER (O.).

RASCHEVSKY (K.). Elementary algebra. Textbook for higher schools. (In Russian.) Moscow, 1912. 8vo. 301+3 pp.

SALOMON (A.). Leçons d'algèbre. 7e édition. Paris, Vuibert, 1912. 16mo. 267 pp.

SCHIFFNER (F.). See JACOB (J.).

SCHULTE-TIGGES (A.). See MEHLER (F. G.).

SCHULTZE (A.). Advanced algebra. 11th reprint. New York, Macmillan, 1913. 12mo. 562 pp. Half leather. $1.25

SCHWATT (I.). See FISHER (G. E.).

SOCCI (A.). See BERTRAND (G.).

TRAVNIČEK (J.). See JACOB (J.).

VAN DER HEYDEN (A. F.). Notes on algebra. Middleborough, W. Appleyard, 1912. 8+133 pp.

III. APPLIED MATHEMATICS.

ARMSTRONG (H. F.). Descriptive geometry for students of engineering science and architecture. New York, Wiley, 1915. 8vo. 6+125 pp. $2.00

BERLINER (S.). Renten und Anleihen. Leipzig, C. E. Poeschel, 1912. 8vo. 10+142 pp.

BRAGG (W. H. and W. L.). X rays and crystal structure. London, Bell, 1915. 8vo. 7s. 6d.

BRAGG (W. L.). See BRAGG (W. H.).

CALENDARIO astronomico delle colonie italiane d'Affrica per l'anno 1915. (Istituto geografico militare.) Firenze, tip. Barbèra, di Alfani e Venturi, 1914. 8vo. 73 pp.

CALENDARIO del r. osservatorio astronomico di Napoli per l'anno 1915. Napoli, 1915. 16mo. 75 pp.

DRURY (F. E.). Geometry of building construction. Second year course. London, Routledge, 1915. 8vo. 3s.

EDWARDS (E. J.) and TICKLE (M. J.). Practical science and mathematics for the second year preliminary technical or industrial course. London, Routledge, 1915. 8vo. 8+175 pp. 1s. 6d.

ELDERTON (W. P.) and FIPPARD (R. C.). The construction of mortality and sickness tables. London, Macmillan, 1914. 12mo. 120 pp. $0.80

FERRARIO (L.). Studî di meccanica molecolare. Milano, Hoepli, 1915. 8vo. 165 pp. L. 4.50

FIPPARD (R. C.). See ELDERTON (W. P.).

FOSCHI (R. C.). Una facile deduzione del teorema dei tre momenti. Città di Castello, S. Lapi, 1914. 8vo. 8pp.

LOMBARDI (L.). Corso teorico-pratico di elettrotecnica. 2a edizione. Vol. II. Milano, 1914. 8vo. 12+718 pp.+5 tables. L. 20.00

MAGGI (G. A.). Geometria del movimento. Lezioni di cinematica con un' appendice sulla geometria della massa. 1915. 8vo. L. 8.00

TICKLE (M. J.). See EDWARDS (E. J.).

TWENTY-FOURTH ANNUAL LIST OF PAPERS

READ BEFORE THE AMERICAN MATHEMATICAL SOCIETY AND SUBSEQUENTLY PUBLISHED, INCLUDING REFERENCES TO THE PLACES OF THEIR PUBLICATION.

ALEXANDER, J. W., II. A proof of the invariance of certain constants of analysis situs. Read Sept. 8, 1913. *Transactions of the American Mathematical Society*, vol. 16, No. 2, pp. 148–154; April, 1915.

ARCHIBALD, R. C. Euclid's Book On Divisions of Figures (*περὶ διαιρέσεων βιβλίον*) with a restoration based on Woepcke's text and the Practica Geometriæ of Leonardo Pisano. Read April 25, 1914. Cambridge, University Press, 1915. 8+88 pp.

BARUS, C. The mathematician in modern physics. Read Sept. 8, 1914. *Science*, new ser., vol. 40, No. 1038, pp. 721–727; Nov. 20, 1914.

BATEMAN, H. The quartic curve and its inscribed configurations. Read Feb. 22, 1913. *American Journal of Mathematics*, vol. 36, No. 4, pp. 357–386; Oct., 1914.

—— The structure of the æther. Read Feb. 28, 1914. *Bulletin of the American Mathematical Society*, vol. 21, No. 6, pp. 299–309; March, 1915.

—— The mathematical analysis of electrical and optical wave-motion on the basis of Maxwell's equations. Read (Chicago) April 29, 1911 and Feb. 22, 1913. Cambridge, University Press, 1915. 6+159 pp.

BAUER, G. N., and SLOBIN, H. L. Algebraic and transcendental numbers. Read (Chicago) Dec. 27, 1913. *Rendiconti del Circolo Matematico di Palermo*, vol. 38, No. 3, pp. 353–356; Nov.–Dec., 1914.

BELL, E. T. An arithmetical theory of certain numerical functions. Part 1. Read Oct. 26, 1912 and (San Francisco) May 22, 1914. *University of Washington Publications (Science)*, 1915. 47 pp.

BERNSTEIN, B. A. A complete set of postulates for the logic of classes expressed in terms of the operation "exception," and a proof of the independence of a set of postulates due to Del Ré. Read (San Francisco) Oct. 25, 1913. *University of California Publications in Mathematics*, vol. 1, No. 4, pp. 87–96; May 15, 1914.

BIRKHOFF, G. D. An elementary double inequality for the roots of an algebraic equation having greatest absolute value. Read April 24, 1915. *Bulletin of the American Mathematical Society*, vol. 21, No. 10, pp. 494–495; July, 1915.

BLICHFELDT, H. F. A new principle in the geometry of numbers, with some applications. Read (San Francisco) April 12, 1913. *Transactions of the American Mathematical Society*, vol. 15, No. 3, pp. 227–235; July, 1914.

BLISS, G. A. A note on symmetric matrices. Read (Chicago) April 10, 1914. *Annals of Mathematics*, ser. 2, vol. 16, No. 1, pp. 43–44; Sept., 1914.

—— The Weierstrass *E*-function for problems of the calculus of variations in space. Read (Chicago) Dec. 26, 1913. *Transactions of the American Mathematical Society*, vol. 15, No. 4, pp. 369–378; Oct., 1914.

—— Generalizations of geodesic curvature and a theorem of Gauss concerning geodesic triangles. Read April 28, 1906. *American Journal of Mathematics*, vol. 37, No. 1, pp. 1–18; Jan., 1915.

—— A note on functions of lines. Read (Chicago) Dec. 28, 1914. *Proceedings of the National Academy of Sciences*, vol. 1, No. 3, pp. 173–177; March, 1915.

BLISS, G. A., and UNDERHILL, A. L. The minimum of a definite integral for unilateral variations in space. Read (Chicago) Dec. 27, 1913. *Transactions of the American Mathematical Society*, vol. 15, No. 3, pp. 291–310; July, 1914.

BÔCHER, M. On a small variation which renders a linear differential system incompatible. Read April 25, 1914. *Bulletin of the American Mathematical Society*, vol. 21, No. 1, pp. 1–6; Oct., 1914.

—— The smallest characteristic numbers in a certain exceptional case. Read Sept. 8, 1914. *Bulletin of the American Mathematical Society*, vol. 21, No. 1, pp. 6–9; Oct., 1914.

BUCHANAN, D. Periodic orbits on a smooth surface. Read (Chicago) Dec. 26, 1913. *American Journal of Mathematics*, vol. 37, No. 1, pp. 79–94; Jan., 1915.

BURNS, J. E. The abstract definitions of groups of degree 8. Read Sept. 8, 1913. *American Journal of Mathematics*, vol. 37, No. 2, pp. 195–214; April, 1915.

CAJORI, F. The history of Zeno's arguments on motion: phases in the development of the theory of limits. Read Sept. 9, 1913 and (Chicago) April 11, 1914. *American Mathematical Monthly*, vol. 22, No. 1, pp. 1–6; Jan., 1915: No. 2, pp. 39–47; Feb., 1915: No. 3, pp. 77–82; March, 1915: No. 4, pp. 109–115; April, 1915.

CARMICHAEL, R. D. On series of iterated linear fractional functions. Read Sept. 8, 1913. *American Journal of Mathematics*, vol. 36, No. 3, pp. 267–288; July, 1914.

CARMICHAEL, R. D., and MASON, T. E. Note on the roots of algebraic equations. Read (Chicago) April 10, 1914. *Bulletin of the American Mathematical Society*, vol. 21, No. 1, pp. 14–22; Oct., 1914.

CHITTENDEN, E. W. The converse of the Heine-Borel theorem in a Riesz domain. Read (Chicago) April 11, 1914. *Bulletin of the American Mathematical Society*, vol. 21, No. 4, pp. 179–183; Jan., 1915.

—— Infinite developments and the composition property $(K_{12}\, B_1)$ in general analysis. Read (Chicago) April 6, 1912. *Rendiconti del Circolo Matematico di Palermo*, vol. 39, No. 1, pp. 81–108; Jan.–Feb., 1915.

COBLE, A. B. Restricted systems of equations (second paper). Read Dec. 31, 1913. *American Journal of Mathematics*, vol. 36, No. 4, pp. 395–418; Oct., 1914.

—— Point sets and allied Cremona groups. Read April 25, 1914. *Transactions of the American Mathematical Society*, vol. 16, No. 2, pp. 155–198; April, 1915. *Proceedings of the National Academy of Sciences*, vol. 1, No. 4, pp. 245–248; April, 1915.

COLE, F. N. Note on solvable quintics. Read Jan. 2, 1915. *Bulletin of the American Mathematical Society*, vol. 21, No. 9, pp. 462–464; June, 1915.

CONNER, J. R. The rational sextic curve, and the Cayley symmetroid. Read Dec. 31, 1912. *American Journal of Mathematics*, vol. 37, No. 1, pp. 29–42; Jan., 1915.

COPELAND, L. P. On the theory of invariants of n-lines. Read Feb. 22, 1913. *Annals of Mathematics*, ser. 2, vol. 16, No. 1, pp. 7–14; Sept., 1914.

CUMMINGS, L. D. On a method of comparison for triple systems. Read April 25, 1914. *Transactions of the American Mathematical Society*, vol. 15, No. 3, pp. 311–327; July, 1914.

CRAIG, C. F. See SHARPE, F. R.

DECKER, F. F. On the order of a restricted system of equations. Read Dec. 28, 1910. *American Journal of Mathematics*, vol. 37, No. 2, pp. 159–178; April, 1915.

DICKSON, L. E. Modular invariants of the system of a binary cubic, quadratic, and linear form. Read Sept. 8, 1914. *Quarterly Journal of Pure and Applied Mathematics*, vol. 45, No. 4, pp. 373–384; Sept., 1914.

—— Invariants in the theory of numbers. Read Sept. 8, 1914. *Transactions of the American Mathematical Society*, vol. 15, No. 4, pp. 497–503; Oct., 1914.

—— The points of inflexion of a plane cubic curve. Read Sept. 8, 1914. *Annals of Mathematics*, ser. 2, vol. 16, No. 2, pp. 50–66; Dec., 1914.

—— Linear algebras. Read (Chicago) Dec. 27, 1913. Cambridge Tracts in Mathematics and Mathematical Physics, No. 16. Cambridge, University Press, 1914. 8+73 pp.

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CONSTITUTION.

ARTICLE I.

This Society shall be called the AMERICAN MATHEMATICAL SOCIETY.

ARTICLE II.

The object of the Society shall be to encourage and maintain an active interest in mathematical science.

ARTICLE III.

The officers of the Society shall be a President, two Vice-Presidents, a Secretary, a Treasurer, a Librarian, and a Committee of Publication to consist of three members who at the same time may hold individually any of the other offices.

ARTICLE IV.

The officers of the Society, the ex-Presidents, the Editorial Committee of the Transactions, the Secretary of the Chicago Section, and twelve other members of the Society shall constitute a Council, which shall have general charge of the affairs of the Society.

ARTICLE V.

1. The officers of the Society and four other members of the Council shall be elected by ballot at the Annual Meeting of each year. An official ballot shall be sent to each member at least one month prior to the Annual Meeting, and such ballots, if returned to the Secretary in envelopes bearing the names of the voters, shall be counted at the Annual Meeting. Each such ballot shall contain a name proposed by the Council for each vacancy, with blank spaces in which the voter may substitute other names. A majority of all votes cast in person or by mail shall be necessary to election. In case of failure to secure a majority for any office, the members present at the Annual Meeting shall choose by ballot between the two having the highest number of votes. The members of the Editorial Committee of the Transactions shall be appointed by the Council.

2. The ex-Presidents of the Society shall be permanent members of the Council; the term of office otherwise shall be two years for the President, one year for other officers, and three years for other mem-

bers of the Council, and until their successors shall be elected. For election as President shall be eligible those members of the Society who have served in the office of Vice-President and who are not holding office as President or Vice-President; and for election as Vice-President shall be eligible those members of the Society not holding office as President or Vice-President. No elected member of the Council, having completed a term of three years, shall be reëlected until at least one year shall have intervened.

3. If the President of the Society die or resign before the expiration of his term of office the Council may designate one of the Vice-Presidents to serve as Acting President until the next Annual Meeting, when a President shall be elected by the Society. Such vacancies as may exist at any time among the other offices may be filled by the Council. If any elected member of the Council die or resign more than one year before the expiration of his term, the vacancy for the unexpired term shall be filled by the Society at the next Annual Meeting.

Article VI.

This Constitution shall not be amended unless notice of the proposed amendment be sent by mail to every member of the Society at least four weeks in advance of the meeting at which such amendment is to be considered; and such amendment in order to be adopted must receive two-thirds of the votes cast in person and by mail.

BY-LAWS.

I.

The election of new members of the Society shall be by vote of the Council. Application for admission shall be made by the applicant personally, on blanks provided by the Secretary, and shall be endorsed by two members of the Society. Such applications, being received at any meeting of the Council, shall be laid over for action at any subsequent meeting.

II.

1. Persons elected by the Council, as provided in By-Law I, shall be admitted to membership in the Society upon the payment, within sixty days of the date of their election, of an initiation fee of five dollars.

2. The annual dues shall be five dollars, payable on the first of January. Each new member shall pay in proportion to the unexpired fraction of the year at the time of his election. Should the annual

dues of any member remain unpaid beyond a reasonable time, the Council shall remove his name from the list of members, after due notice.

3. On the payment of fifty dollars in one sum, any member of at least four years standing and not in arrears of dues may become a life member and shall thereafter be exempt from all annual dues.

III.

The times and places of meetings of the Society shall be designated by the Council, but the Annual Meeting shall be held on a date between the twenty-sixth and thirty-first of December, inclusive; and Regular Meetings shall be held on the last Saturday of February, April, and October, unless otherwise ordered by the Council. Notice of the time and place of each meeting shall be sent by the Secretary to all members of the Society.

IV.

Whenever it shall appear to the Council that a sufficient number of members of the Society are desirous of conducting in any locality periodic meetings for the reading and discussion of mathematical papers, the Council may authorize the formation of a Section to be composed at each sectional meeting of such members of the Society as may be present; and the Council shall have the right to withdraw such authorization.

V.

Papers intended for presentation at any meeting or sectional meeting for the Society shall be passed upon in advance of the meeting by a programme commitee appointed by or under the authority of the Council; and only such papers shall be presented as shall have been approved by such committee. Papers in form unsuitable for publication, if accepted for presentation, shall be referred to on the programme as preliminary communications or reports.

VI.

1. The order of business at the meetings of the Society shall be as follows:

1. Reading of the minutes.
2. Recommendations and reports.
3. Elections.
4. Miscellaneous business.
5. Presentation and discussion of papers previously approved by the programme committee.

2. No question relative to admistration shall be considered at any meeting except the Annual Meeting without the recommendation of the Council.

VII.

It shall be the duty of each President to deliver an address before the Society at the Annual Meeting at which his term of office expires.

VIII.

1. The President may convoke the Council whenever the affairs of the Society require it.

2. A request in writing from two members of the Council shall render the convocation obligatory.

IX.

1. The Society shall publish a periodical of critical and historical nature called the BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY.

2. The BULLETIN shall be in charge of the Committee of Publication, which may associate with itself other members of the Society whose editorial assistance it may require.

X.

1. The Society shall publish a periodical which shall have for its object to make known, as widely as possible, the more important researches presented at the meetings of the Society. This periodical shall be called the TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY.

2. The editorial management of the TRANSACTIONS shall be entrusted to a committee of three editors, to be appointed by the Council. One editor shall be appointed annually at the April meeting, to serve for three years. Each editor shall begin his term of office on the first day of October of the year in which he is appointed. Members of the committee shall be eligible for reappointment. If a member of the commitee die or resign before the completion of his term of office, the vacancy shall be filled for the unexpired term by the Council.

3. The editorial committee shall have power to elect one of its members as editor-in-chief; but if at any time the committee shall report to the Council that it has been unable to agree upon an editor-in-chief, the Council shall designate one of the members of the committee to act as editor-in-chief. The committee shall have power to associate with itself other members of the Society whose editorial assistance it may require.

4. The TRANSACTIONS shall not be distributed free to the members of the Society; but members of the Society shall have the right to subscribe to the TRANSACTIONS at three-fourths of the advertised price.

XI.

No by-law shall be enacted, amended or suspended, except by a two-thirds vote of the members present at a meeting of the Society, and upon the recommendation of the Council.

REPORT OF THE TREASURER FOR THE YEAR 1914

RECEIPTS.

Balance from 1913	$ 9,153.58
Annual dues for 1912	150.00
Annual dues for 1913	569.50
Annual dues for 1914	2,587.88
Annual dues for 1915	40.95
Dues from new members for parts of 1913	0.83
Dues from new members for parts of 1914	76.67
Initiation fees for 1914	200.00
Life membership fees	200.00
Sale of reprints	137.02
Royalties	10.95
Sale of Evanston Colloquium Lectures	4.80
Sale of Princeton Colloquium Lectures	98.30
Sale of Madison Colloquium Lectures	166.05
Sale of Bulletin	526.08
Sale of Transactions	1,037.49
Interest on investments	372.50
Advertisements in Bulletin	216.00
Insurance	847.06
Exchange	1.87
	$16,397.53

DISBURSEMENTS

Secretary's Office	$ 836.23
Treasurer's Office	123.54
Library	175.80
Committee of Publication	138.04
Editorial Committee of Transactions	202.61
Bulletin (ten numbers and index vols. 11–20)	1,829.86
Transactions (four numbers)	1,724.59
List of Officers and Members	110.00
Chicago Section	36.17
San Francisco Section	14.50
Southwestern Section	36.47
Princeton Colloquium Lectures	9.35
Madison Colloquium Lectures	755.92
Salary of Clerk	720.00
Euler Subscription	193.78
Insurance	27.22
Exchange	1.70
	$ 6,935.78

Investments (par value $8,500.00)	$8,259.50	
Balance in First National Bank (Ithaca)	1,202.25	$9,461.75
		$16,397.53

The Life Membership Fund now amounts to $5,198.85.

J. H. TANNER,
Treasurer.

ITHACA, N. Y.,
December 23, 1914.

REPORT OF THE AUDITING COMMITTEE

The undersigned Auditing Committee, appointed by the American Mathematical Society, have this day examined the accounts of the Treasurer, compared them with the vouchers, so far as not destroyed by the fire in the Secretary's office, with the bank balance, and with the list of securities, and find the following to be correct:

Balance from December 23, 1913		$ 9,153.58
Cash receipts since December 23, 1913		7,243.95
		$16,397.53
Balance in First National Bank (Ithaca)	$1,202.25	
Investments (par value $8,500.00)	8,259.50	
Disbursements since December 23, 1913	6,935.78	
		$16,397.53

H. E. HAWKES,
W. M. STRONG,
Auditing Committee.

NEW YORK, N. Y.,
December 23, 1914.